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# Analytical Modelling and Performability Evaluation of Multi-Channel WLANs with Global Failures

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**Abstract:** Wireless local area networks (WLANs) which are based on IEEE 802.11 standard are used widely in existing local area network configurations. IEEE 802.11 offers multiple non-overlapping channels to increase the capacity of the network. There are strong evidences that WLANs are prone to impairments. In order to improve the quality of service (QoS) and to evaluate the performance of WLANs realistically, the availability of the systems should be considered. This paper studies performability evaluation of a multi-channel WLAN using analytical modelling approach. Unlike the existing studies, the failures of the overall system, where a critical function unit fails making all the channels unavailable are considered. A new term is introduced as global failures. It is possible to solve the models considered using matrix geometric method where system parameters and minimal non negative solution $R$ is computed by an iterative method. However spectral expansion method is a well-known alternative where the iterative calculations for solving $R$ is avoided using eigenvalues and eigenvectors. The exact spectral expansion method is employed to obtain performability measures such as mean queue length and blocking probability. Iterative refinements are employed in solution of simultaneous equations.

**Keywords:** Analytical modelling, 2-dimensional Markov chain analysis, performability evaluation, availability, spectral expansion, multi-channel WLANs.

## 1 Introduction

The demand of new multimedia services, increase in bandwidth available to users, high traffic densities, and ubiquitous connectivity have caused the rapid growth of wireless and mobile communication systems. Wireless LANs (WLANs) based on the IEEE 802.11 standard have gained widespread popularity mainly because they provide users unlimited access to the Internet with relatively high data rates. The IEEE 802.11 specifications provide a multi-rate capability at the physical layer (PHY) to accommodate mobile users with diverse wireless channel conditions [1]. The IEEE 802.11b specification supports data rate up to 11 Mbps. The subsequent versions of WLANs such as 802.11a, 802.11g, 802.11e and the latest version, 802.11n, offers maximum rate of 54 Mbps and 500-600 Mbps respectively [2,3]. The classical IEEE 802.11 MAC protocol considers a single shared channel. Nevertheless, multiple channels can also be used [1,5,20,22,24]. The recent WLANs based on IEEE 802.11 standards permit simultaneous operation on multiple non-overlapping channels at the physical layer. Extensive research has been carried out on multi-channel MAC protocols for IEEE 802.11 based WLANs [1, 21, 23, 24]. These approaches proposed multi-channel WLANs to improve overall network performance depending on different channel assignment schemes. In other words, the wireless access points (APs) can be considered as multi-channel systems with multiple wireless radios each operating independently on different channels. These multiple radios and multiple channels facilitates dynamic selection of channels for the users to transmit and receive packets without interfering each other [1,2,4,5,8,14]. Thus, these new features improve the QoS such as throughput, blocking probability, response time etc. [15, 21–24].

In [20] two cross-layer models have been proposed to access transmissions channels. The first model is single class type traffic. The traffic divided into two classes as high and low priority traffic in the second model. The performance parameters for both single and multi-channel wireless networks, like the requests throughput, acceptance probability are calculated. WLANs suffer especially when the users with different and high transmission rates share the same physical channel. In order to solve this problem different transmission channels have been proposed in [21] to achieve better QoS. In addition, an 802.11 based multi-hop wireless ad-hoc network architecture have been proposed in [22] where it employs multiple radio channels simultaneously in the system. Multiple channels along with the bi-directional channel reservation policy of ad hoc networks based on IEEE MAC protocol considered is considered in [23] using analytical modelling. The paper also considered two different channel schemes for further analysis. In [24] existing multi-channel protocols are proposed to enhance the QoS by classifying them into single-radio protocols, multi-radio single-hop protocols, and multi-radio multi-hop protocols. The existing studies in the literature propose modified MAC layer architecture which supports multi-channel networks. The main aim is to find an optimal channel for a single packet transmission to improve QoS.

In order to improve QoS and to maximize the performance of such systems analytical modelling and evaluation is still one of the key issues in the performance characterization of wireless communication systems. Various approaches exist in the literature for the performance evaluation of WLANs [6–8, 12, 14, 20]. However, most of the existing work has been done on pure performance analysis of IEEE 802.11 distributed coordination function (DCF) as well as the analysis of the IEEE 802.11 MAC protocol. The traditional pure performance models are too optimistic. They ignore failure and recovery behaviour and only focus on system's ability to perform. System availability models, that consider failure and repair, should be taken into account in order to obtain more realistic QoS measures. The performability modelling and evaluation is a combination of performance and availability models [9–11, 13]. In practice, WLANs are prone to system failures such as failure of an AP and base station (BS) like any other physical system [10, 11, 13, 15, 16, 19]. The signal strength, fading, reflection and refraction of signals, and man-made noise etc. are some important factors that can be the cause of the channel unavailability in WLANs [5,10,11,13,15]. In WLANs, failing of AP/BS and/or the components of networks may cause the failure of all the available channels. This fault is termed as global system failure of WLAN in this paper. Examples of system failure scenarios in WLANs would include failure of APs and/or BSs, loss of the link between APs and users etc. This type of failures may occur when an AP fails or the communication links between an AP and service providers do not function, or operate. Failures may occur due to software, hardware, human error, or a combination of these factors in WLANs. Different kind of failures, reasons, and failure affects can be found in [5, 9, 11–13, 16, 19]. Channel failures due to the environmental interference as well as from other wireless devices are considered in [15]. Moreover, in [13], channels are defined as virtual channels and each channel is subject to failure for transmission because of multipath fading in the WLANs.

In case of a system failure, all the channels, resources, services, and the components become unavailable to the users. System and channel failures occurring in system is lead to degradation of the entire system performance. However, the existing studies do not consider the channel unavailability with global failures which can be quite critical for QoS measurements. One of the main objectives in this paper is to model and analyse the system performance considering global failures and recovery in multi-channel WLANs. Thus, modelling and analysis of the system performance with global failures and repairs is considered with potential channel failures in multi-channel WLANs. In addition, mobility related factors are considered and analysed. Therefore the proposed model attempts to understand, and improve system performance in terms of system capacity, mobility and blocking probability of packets. The channel assignment scheme employed is another major issue in the performance characterisation of wireless communication systems. There have been extensive studies on dynamic channel access in multi-channel WLANs recently [1, 3, 4, 14, 20, 22, 23]. This is because dynamic channel access scheme is appropriate and efficient scheme to improve QoS of multi-channel WLANs. There is no fixed relationship between channels and systems in dynamic channel access. All channels are assigned into a central pool (e.g bandwidth). The channels are assigned to the incoming packet requests dynamically and channel returns to the central pool after the service [3, 4]. In order to meet the future demands and increase available bandwidth for each user within WLANs, dynamic channel access is also taken into consideration in this paper. This is done by (assuming the packets are accepted with certain bandwidth assigned) optimal allocation of WLAN bandwidth for the number of channels required. All the channels used by the same WLAN are perfectly coordinated and synchronised.

Spectral expansion method is employed for the exact solution of the analytical model considered [9, 10]. Using the proposed model and an exact solution, important performability measures, such as mean queue length and blocking probability can be computed.

The rest of the paper is organized as follows: Section II describes the proposed model. The two dimensional modelling approach and exact steady state solution are explained in section III and section IV respectively. In section V, numerical results computed by using an exact solution approach are presented. Some pure performance results are also presented for comparison. Conclusions and recommendations are provided in section VI.

## 2 The System Considered and The Analytical Model

The WLAN presented is subject to failures. The entire system failures (e.g AP or BS) are considered as global failures that may be caused by power cut-off or resetting AP etc. The model covers mobility issues and various queue capacities as well. The system has multiple identical channels. Allocation of packets is usually done by availability of channels and in this regard, it is well known that, in terms of efficiency the common queue is more suitable than individual queues in the queuing theory [7, 10, 11, 17]. The proposed model considered for performability evaluation of multi-channel WLANs is shown in Fig. 1.

The system consists $S$ identical channels, numbered 1, 2, 3, $\cdots$, $S$ with a common queue. The common queue is bounded with a capacity of $W$ ($W \geq S$). The maximum number of calls in the system is equal to the number of calls assigned to the channels plus the queuing capacity. This is given by $L$ where, $L = S{+}W$. The superposition of all arrival streams (potential arrivals from other wireless technologies can be incorporated) follows a Poisson process. In other words, inter-arrival times of the incoming call requests are assumed to follow an exponential distribution with mean rate of $\lambda$ similar to studies [7–11], [17]. If the channels are available and idle in the

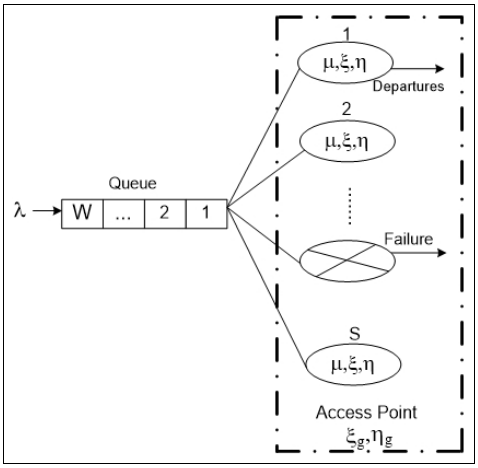

Figure 1: The queueing system considered for proposed multi-channel WLAN

WLAN, the packet arrivals can be assigned to any channel. Otherwise, in case of a channel failure, or a busy channel, the incoming packet request is queued. Similarly, the service times of the packet serviced by channel $s$ ($s$=1, 2, $\cdots$ , $S$) are distributed exponentially. In addition to that, the dwell time, which is the time that mobile users spend in the WLAN coverage area, is assumed to be distributed exponentially with mean $1/\mu_{wd}$ [9, 10]. Equation (1) is used to calculate $\mu_{wd}$.

$$\mu_{wd} = \frac{E[v]F}{\pi O} \tag{1}$$

Where, $E[v]$ denotes the expected value of the random variable $v$, which is the average speed of the mobile users. Since the coverage area of the WLANs are relatively small and most of the users are almost static, $v$ is taken only for pedestrians (e.g $E[v]$=2km/h). $F$ is the length of the perimeter of cell (a WLAN with an arbitrary shape is assumed), and $O$ is the area of the WLAN [9], [10], [11]. $T_{ch}$ is the average packet holding time in the WLAN and follows an exponential distribution with mean $E[T_{ch}]$=$1/\mu_{ch}$. Since $F$ is small for WLAN, it would be impractical to consider high speeds as the user will be out of range before they are able to use the connection. Hence in this study, near stationary speeds are considered. $T$ can be defined as the total channel holding time of a packet, which is exponentially distributed with mean E[$T$] = $1/\mu$, where

$$\mu = \mu_{ch} + \mu_{wd} \tag{2}$$

Channel assignment schemes can be implemented in many different ways depending on the medium access control (MAC) and the system architecture. WLAN (the IEEE 802.11) MAC assures fair sharing of WLAN bandwidth/channels stations since WLANs use a shared medium [1, 4, 6, 7, 14, 19, 23]. Traffic load makes channels unreliable and unavailable for transmission. This causes severe degradation in the performance of WLANs. In order to solve the performance degradation problem in WLANs, frequency channels should be assigned to APs in an appropriate and efficient manner. Hence, in this paper, an analytical model developed maximizes the bandwidth usage according to the number of mobile users within the WLAN.

In other words, considering the behaviour of MAC mechanism, the users fairly share WLAN bandwidth. This sharing can be thought as a dynamic bandwidth allocation in WLAN. This is done by (assuming the call is accepted with certain bandwidth assigned) optimal allocation of WLAN bandwidth for the number of channels required. The main objective of using dynamic bandwidth allocation is to improve the overall QoS of an entire WLAN by sharing the available bandwidth. When dynamic channel allocation scheme is employed, the channel does not change for different numbers of packets in the performance model employed.

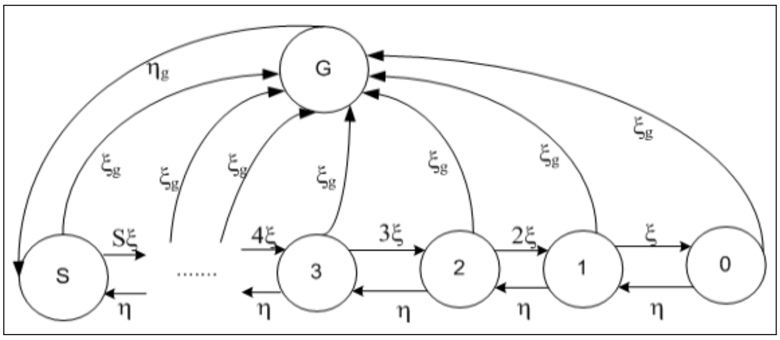

Figure 2: The operative states of the proposed system

The failure and repair behaviour of each channel are also represented by exponentially distributed failure and repair times with means $1/\xi$ and $1/\eta$, respectively [?], [16]. A single repairman facility is assumed for all of the channels but the models provided can easily be extended for systems with multiple repairmen facilities. An inoperative period of a channel would also include the possible waiting time for a repairman. No operative channel can be idle if there are packets waiting, and no repairman can be idle if there are failed channels waiting for repair. On the other hand, the system (AP) can fail, and the repair facility is provided with means $1/\xi_g$ and $1/\eta_g$, respectively. In case of global failures ($\xi_g$ is the respective global failure rate) the priority of repair is given to the AP because the channels cannot be used when the AP is inoperative. All inter-arrival, service, failure/repair of channels and global failure and repair time variables are distributed exponentially and are independent of each other. The operative state of such a system is illustrated in Fig. 2. The states 1, 2, 3, $\cdots$ , $S$ are the working states of the multi-channel WLAN. In state 0 there are no channels available due to the channel failures and in state $G$ there are no channels available due to the AP failures.

## 3 Two Dimensional Markov Representation of The Proposed Model

The state of the system at time $t$ can be described by a pair of integer valued random variables, $I(t)$ and $J(t)$, specifying the channels plus global failure/recovery configurations and the number of packets present, respectively. Here, channel configuration, and hence the range of $I(t)$, refers to the operative states of the channels. In general, there are $S+2$ configurations, represented by the values $I(t) = 0,\ 1,\ \cdots,\ S,\ G$. The first $S+1$ configurations represent the number of available channels in the system (from 0 to $S$). The (S+2)$^{th}$ state is used to represent the system in case the AP is down (state $G$). When the AP is repaired with repair rate $\eta_g$, the system resets with $S$ operative channels. $J(t)$ is the number of jobs in the system, $J(t) = 0$, 1, $\ldots$, $L$. $Z = [I(t), J(t)]$; $t \geq 0$ is an irreducible Markov process on a lattice strip (a QBD process), that models the system. Its state space is, (0, 1, $\cdots$, $S+1$) x (0, 1, $\cdots$, $L$).

Let the possible operative states of both channels and AP be represented in the horizontal direction (the number of channels available and $G$ for the AP) and the number of packets in the vertical direction of a lattice strip. Here, $A$ is the matrix of instantaneous transition rates from state $(i, j)$ to state $(k, j)$, (i=0, 1, $\cdots$, $S+1$; k=0, 1, $\cdots$, $S+1$; $i \neq k$; j=0, 1, $\cdots$, $L$), with zeros on the main diagonal, caused by a change in the state. These are the purely lateral transitions of the process $Z$. Matrices $B$ and $C$ are transition matrices for one-step upward and one-step downward transitions respectively [9], [10]. $B$ is the matrix of one-step upward transition rate from state $(i, j)$ to state $(k, j+1)$, (i=0,1, $\cdots$, $S+1$; k=0,1, $\cdots$, $S+1$; and j=0,1, $\cdots$, $L$), caused by a packet arrival. Clearly, the elements of $A$ depend on the parameters $S$, $\xi$, $\eta$, $\xi_g$, $\eta_g$. The transition matrices of a system with $S$ channels are of size $(S+2)\times(S+2)$. It is possible to specify the numbering of the matrices as (0, 1, 2, 3, $\cdots$, $S+1$) for states (0, 1, 2, 3,$\cdots$, $S$, $G$) respectively. The state transition matrices $A$, $A_j$, $B$, and $B_j$, can be given as follows:

$$A = A_j = \begin{pmatrix} 0 & \eta & 0 & 0 & 0 & 0 & 0 & \xi_g \\ \xi & 0 & \eta & 0 & 0 & 0 & 0 & \xi_g \\ 0 & 2\xi & 0 & \eta & 0 & 0 & 0 & \xi_g \\ 0 & 0 & 3\xi & 0 & \ddots & 0 & 0 & \xi_g \\ 0 & 0 & 0 & \ddots & 0 & \eta & 0 & \xi_g \\ 0 & 0 & 0 & 0 & (S-1)\xi & 0 & \eta & \xi_g \\ 0 & 0 & 0 & 0 & 0 & S\xi & 0 & \xi_g \\ 0 & 0 & 0 & 0 & 0 & 0 & \eta_g & 0 \end{pmatrix}$$

$$B = B_j = \begin{pmatrix} \lambda & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & \lambda \end{pmatrix}$$

On the other hand, $C$ is the matrix of one-step downward transition rate from state $(i, j)$ to state $(k, j-1)$, (i=0, 1, $\cdots$, $S+1$; k=0, 1, $\cdots$, $S+1$; and j=0, 1, $\cdots$, $L$), caused by the departure of a serviced packet. The transition rate matrices do not depend on j for $j \geq M$, where $M$ is a threshold having an integer value. The elements of $C$ and $C_j$ depend on the parameter $\mu$ and can be given as follow:

$$C = C_j = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (S-1)\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & S\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Please note that the stations considered within the coverage are of the WLAN can be mobile. When such a scenario is considered, the transition rate matrix $C$ depends on $j$ since the departure rate caused by mobility depends on the number of packets in the queue [9, 10]. Therefore, the threshold $M$ is taken as $M = L$. If the number of packets in the system is less than the number of available channels, a channel is assigned for each packet. Therefore, the downward transition rate is chosen as the minimum of number of packets and number of available channels. On the other hand, if the number of packets is greater than the number of available channels, all of the available channels are assigned to incoming calls and the calls in the queue have the departure rate $\mu_{wd}$ [9, 10] due to mobility. The matrix $C$ is defined below, together with $C_j$ matrices for two different regions explained above:

$$C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu + W\mu_{wd} & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & (S-1)\mu + W\mu_{wd} & 0 & 0 \\ 0 & 0 & 0 & 0 & S\mu + W\mu_{wd} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

for j > S

$$C_j = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu + (j-S)\mu_{wd} & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & (S-1)\mu + (j-S)\mu_{wd} & 0 & 0 \\ 0 & 0 & 0 & 0 & S\mu + (j-S)\mu_{wd} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

for j ≤ S

$$C_j = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & min(1,j)\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & min(1,j)\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & min(1,j)\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & min(1,j)\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This system can be solved and the steady state probabilities, $p_{i,j}$ can be obtained using the steady state solution presented in the next section.

## 4 Steady State Solution

The solution is given for systems with bounded queueing capacities. The steady-state probabilities of the system considered can be expressed as:

$$p_{i,j} = lim_{t \to \infty} \; P(I(t) = i, J(t) = j); \tag{3}$$

$$0 \leq i \leq S+1, \, 0 \leq j \leq L$$

### 4.1 Spectral Expansion Solution

It is possible to use spectral expansion or matrix-geometric methods to solve the system considered. When matrix-geometric method is employed, a non-linear matrix equation is formed from the system parameters and an iterative method is employed in order to compute the minimal non-negative solution $R$ of this equation. One of the main disadvantages of this method is that it is not possible know the number of iterations needed to compute $R$ for a specified accuracy.

In this study spectral expansion method is employed for steady state solution. Let's define certain diagonal matrices of size $(S+2) \times (S+2)$ as follows:

$$D_j^A(i,i) = \sum_{k=0}^{S+1} A_j(i,k); \quad D^A(i,i) = \sum_{k=0}^{S+1} A(i,k); \tag{4}$$

$$D_j^B(i,i) = \sum_{k=0}^{S+1} B_j(i,k); \quad D^B(i,i) = \sum_{k=0}^{S+1} B(i,k); \tag{5}$$

$$D_j^C(i,i) = \sum_{k=0}^{S+1} C_j(i,k); \quad D^C(i,i) = \sum_{k=0}^{S+1} C(i,k); \tag{6}$$

and $Q_0 = B$, $Q_1 = A - D^A - D^B - D^C$, $Q_2 = C$.
all state probabilities in a row can be defined as:

$$v_j = (p_{0,j}, p_{1,j}, \cdots, p_{S+1,j}); j = 0, 1, 2 \ldots, L \tag{7}$$

The steady-state balance equations for bounded queuing systems ($0 \le j \le L$) can now be written as follows:

$$v_0[D_0^A + D_0^B] = v_0 A_0 + v_1 C_1 \tag{8}$$

$$v_j[D_j^A + D_j^B + D_j^C] = v_{j-1}B_{j-1} + v_j A_j + v_{j+1}C_{j+1}; \tag{9}$$
$$1 \le j \le M-1$$

$$v_j[D^A + D^B + D^C] = v_{j-1}B + v_j A + v_{j+1}C; \tag{10}$$
$$M \le j \le L$$

$$v_L[D^A + D^C] = v_{L-1}B + v_L A \tag{11}$$

The normalizing equation is given as follows:

$$\sum_{j=0}^{L} v_j e = \sum_{j=0}^{L} \sum_{i=0}^{S+1} P(i,j) = 1.0 \tag{12}$$

From the equations above, one can write

$$v_j Q_0 + v_{j+1} Q_1 + v_{j+2} Q_2 = 0 \tag{13}$$
$$(M-1) \le j \le (L-2)$$

Furthermore, the characteristic matrix polynomial $Q(\lambda)$ can be defined as:

$$Q_\lambda = Q_0 + Q_1\lambda + Q_2\lambda^2; \quad \bar{Q}_\beta = Q_2 + Q_1\beta + Q_0\beta^2; \tag{14}$$

where

$$\Psi Q_\lambda = 0; \quad |Q_\lambda| = 0; \quad \phi\bar{Q}_\beta = 0; |\bar{Q}_\beta| = 0; \tag{15}$$

$\lambda$ and $\Psi$ are eigenvalues and left-eigenvectors of $Q_\lambda$ and $\beta$ and $\phi$ are eigenvalues and left-eigenvectors of $\bar{Q}_\beta$, respectively. Note that, $\phi$ is a vector defined as

$$\phi = \phi_0, \phi_1, \ldots, \phi_{S+1} \tag{16}$$

and $\beta$ as:

$$\beta = \beta_0, \beta_1, \ldots, \beta_{S+1}. \tag{17}$$

Furthermore, $v_j = \sum_{k=0}^{S+1}(a_k \Psi_k \lambda_k^{j-M+1} + b_k \phi_k(i) \beta_k^{L-j})$,M-1≤j≤ $L$ and in the state probability form,

$$p_{i,j} = \sum_{k=0}^{S+1}(a_k \Psi_k \lambda_k^{j-M+1} + b_k \phi_k(i) \beta_k^{L-j}) \tag{18}$$

$$M-1 \leq j \leq L$$

Where $\lambda_k (k = 0, 1, \ldots, S+1)$ and $\beta_k (k = 0, 1, \ldots, S+1)$ are $S+2$ eigenvalues each, that are strictly inside the unit circle and $a_k$, $b_k$ (k=0, 1,..., $S+1$) are arbitrary constants which can be scalar or complex-conjugate [9], [10]. All the $a_k$, $b_k$ values and the remaining $v_j$ vectors can be obtained using the process in [9] and [10].

### 4.2 LU Factorization with Partial Pivoting

For the solution of real linear equations to obtain the scalars $a_k$, and $b_k$, (k=0, 1,..., $S+1$) the set of linear equations are considered with a single right-hand side, using an *LU* factorization with partial pivoting, and iterative refinement [25]. LU factorization of the matrix employed is computed first with partial pivoting ($L$ is lower triangular and $U$ is unit upper triangular). An approximation for the answer vector $x$ is then found by forward and backward substitutions. Using additional precision, the residual vector is calculated in turn. An iterative refinement approach is employed until full machine accuracy is obtained.

Once all the steady state probabilities $p_{i,j}$ (i=0, 1, ..., $S+1$; j=0, 1, ..., $L$) are computed, a number of steady-state performance measures can be obtained. For numerical results and discussions, the mean queue length (MQL) and blocking probabilities ($P_B$) of multi-channel WLAN are considered as:

$$MQL = \sum_{j=0}^{L} j \sum_{i=0}^{S+1} p_{i,j} \tag{19}$$

$$P_B = \sum_{i=0}^{S+1} p_{i,L} \tag{20}$$

## 5 Numerical Results and Discussions

Numerical results are presented in this section for the performability analysis of multi-channel WLANs with global failures. The exact spectral expansion solution is employed to obtain performability measures. Numerical results are presented for performability measures of multi-channel WLAN (e.g. MQL and $P_B$). The parameters used are mainly taken from the relevant literature and are $E[T_{ch}] = 60sec$, E[v]=2km/h, R=100m [7, 8, 10, 11, 13, 18].

The results in Fig. 3 show that for low channel failure rates, the system can perform as good as systems without failures. However, as the channel failure rate increases the performance degradation becomes more evident (for $\xi$=0.1, the MQL values become up to twice as much as the MQL values calculated for the other rates). The MQL and $P_B$ measures are presented in Fig. 4 and Fig. 5, respectively. The parameters used are the same as previous computations with various $\lambda$ values.

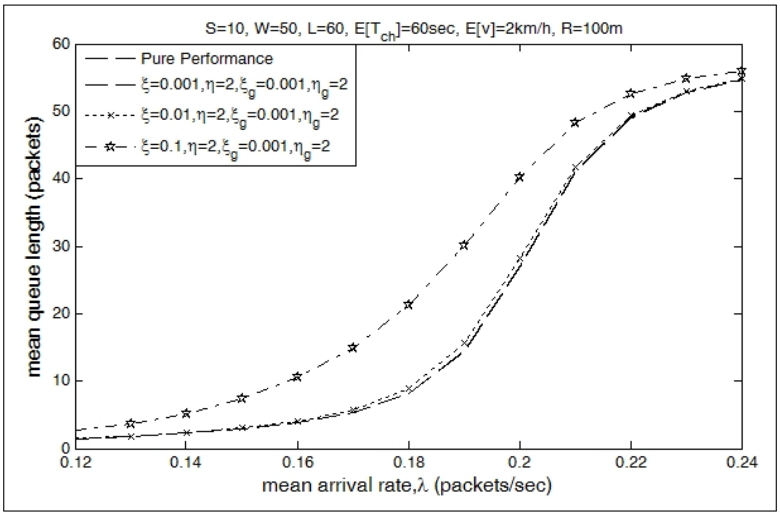

Figure 3: MQL results for systems with various channel failure rates

Fig. 4 and Fig. 5 show that the global failures (AP failures) affect the system significantly. The effects of global failures are more evident for blocking probabilities, since as mean arrival rate and global failure rate increase, the limiting effect of queue capacity becomes more evident. The effects of global failures are considered in Fig. 6 as well. This time a 25 channels system is considered (S=25) together with W=100. MQL values are presented as a function of mean arrivals. The other parameters used are same as previous computations.

Fig. 6 shows that when failure rate of the access point is taken as 0.1 the difference in MQL values is quite significant. However for systems with $\xi_g$=0.01/h and $\xi_g$ =0.001/h the difference is not as significant. This is mainly due to good repair facilities provided. Since the $\eta_g$=2/h the mean repair time is taken as half an hour for the access point. The effects of having various repair times are considered in Fig. 7. The other parameters considered are the same as the ones used for Fig. 6.

The repair time of the access point affects the performance of the system significantly. Fig. 7 clearly shows that systems with mean repair time equal to one hour performs significantly better then systems with $1/\eta_g$=1.5h and $1/\eta_g$=2h. In the model considered the priority of repair is given to the access point. However, since the repair time also includes delay factors such as the time needed for transportation of the repairman, configuration of access point etc., the repair facility should be carefully considered in order to meet expected levels of performability. The results demonstrate that the systems may become overwhelmed because of long repair times,

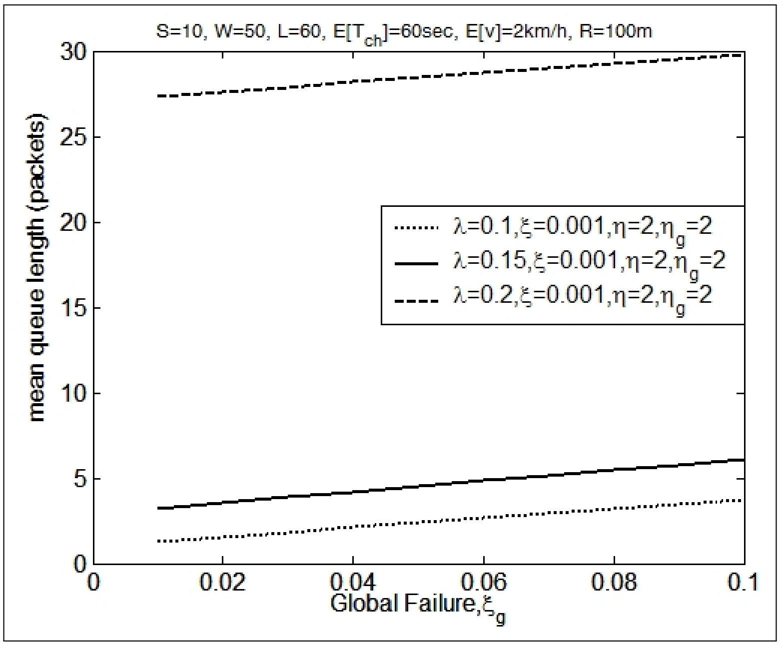

Figure 4: MQL results for systems with various global failure rates

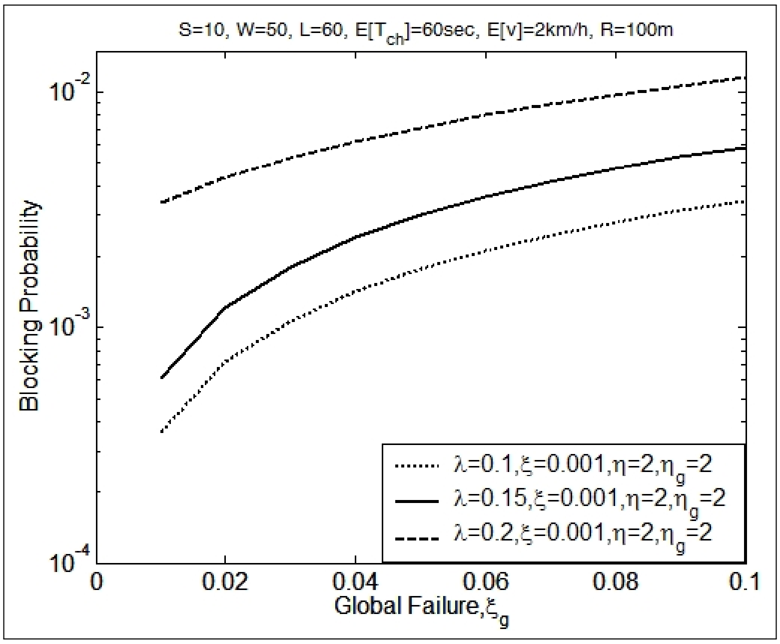

Figure 5: Blocking probabilities for systems with various global failure rates

especially if heavy loads of packet requests are expected.

Fig. 8 shows the effects of having various numbers of channels in a WLAN with various mean incoming arrival rates. The other parameters used are the same as the ones used for Fig. 7. Fig. 8 shows that the number of channels affects the overall performability of the system significantly. When $\lambda$=0.25 is considered, the MQL takes values close to $L$, such as 104.32 for five channel systems and it is around 1.042 for systems with 25 channels. Systems with different number of available channels may perform similarly for heavy loads and light loads, however, it is obvious that as the number of channels employed increases, the system performs better.

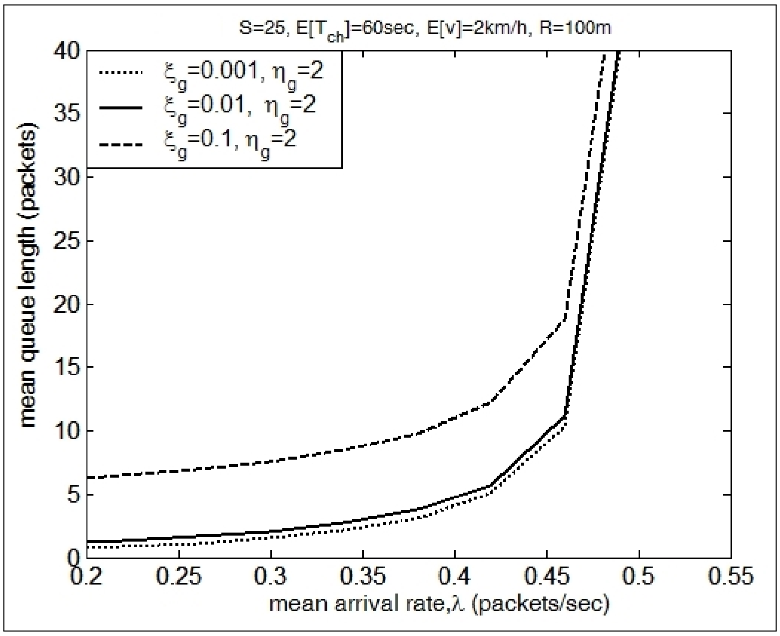

Figure 6: MQL as a function of $\lambda$ for S=25 and W=100 for various $\xi_g$ values

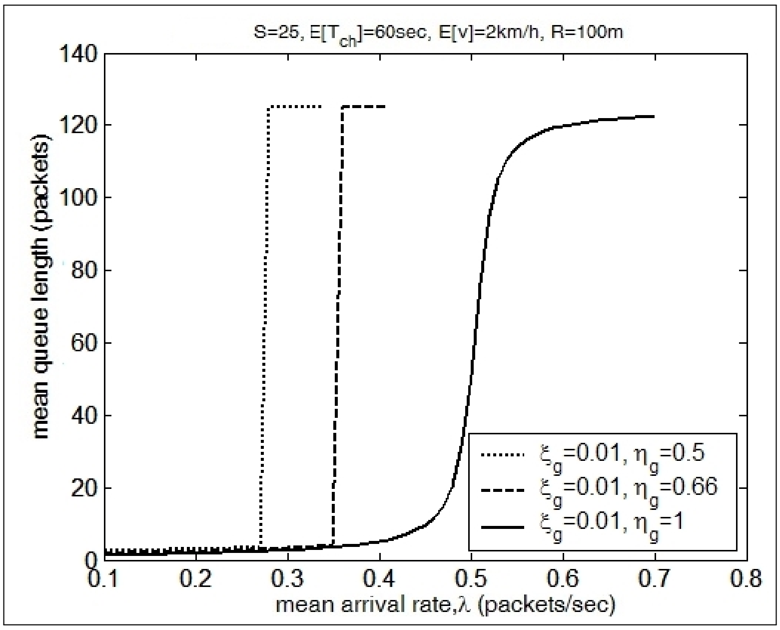

Figure 7: MQL as a function of $\lambda$ for S=25 and W=100 for various $\eta_g$ values

Finally, Fig. 9 shows the MQL as a function of mean rate for incoming packet requests. Two different scenarios are considered for the computations. In the first scenario the mobile stations waiting in the queue for channels are static, whereas in the second scenario the mobile stations are pedestrians with E[v]=2km/h. The other parameters are S=10, W=50, $\xi$=0.01/h, $\eta$=2/h, $\xi_g$=0.001/h, $\eta_g$=2/h. The results show that, although the velocity of pedestrian users is relatively low, in case of high arrival rates, the mean number of users leaving the system due to mobility is quite significant. For example, for $\lambda$=0.2 MQL is 28.3 for systems with static users in the queue, and 9.92 for systems with pedestrian users in the queue.

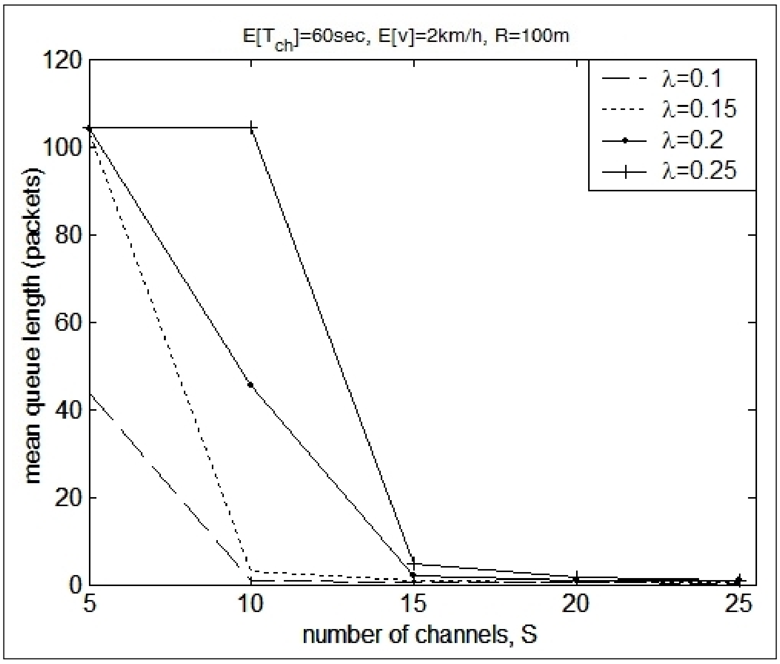

Figure 8: MQL as a function of $S$ for various $\lambda$ values

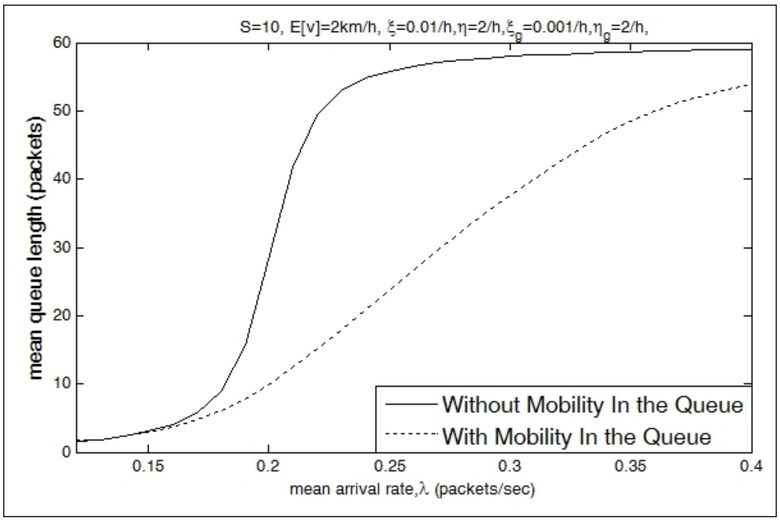

Figure 9: MQL as a function of $\lambda$, for mobile users waiting for service

# 6 Conclusions

In this paper, multi-channel WLAN is considered for performability evaluation. A two dimensional modelling approach and an exact solution approach where iterative refinements are used to increase the accuracy for solution of system of linear equations are employed. The multi-channel failures/repairs, and global failures/repairs of WLANs modelled for exact solution. The dynamic channel allocation scheme is also considered in an attempt to maximise the performance of the underlying infrastructure. The proposed model is used to analyse QoS measures such as mean queue length and blocking probabilities. The presented examples were kept simple due to the introductory nature of the proposed model for the multi-channel WLANs. Obviously, the analysis can be expanded upon for more informed decisions, since the multi-channel WLANs are common today. Recent WLANs have multiple wireless radios each operating independently on different channels. The analysis of the multi-channel WLANs is an important issue in order to achieve better QoS measurements in future wireless and communication systems.

In this paper, in order to obtain more realistic QoS measures, considerable amount of focus is given operative states of multi-channel as well as global failures of WLANs with the proposed analytical model and its exact solution. The results mainly show that, channel and global failures affect the QoS of the system significantly. Therefore, the system availability is important for the future system design and modelling. Results also show that the failures cause significant performance degradations in the system, and the channel and global recovery are important parameters for the system's performance. The systems with higher number of channels perform better as expected. As is widely agreed, future mobile communication systems, fourth-generation (4G) systems, will be heterogeneous networks, which provide ubiquitous access and seamless mobility across heterogeneous network technologies, such as cellular, WLAN, and broadcast access. In 4G wireless networks, mobility management is still one of the most important issues that need to be considered. Thus, results provided in Fig. 9 are very important in that sense.

The results mainly show the analytical model and the exact steady state solution approach presented is very useful in future systems since the recent WLANs technologies operate in multi-channel basis (e.g Hiperlan2 or WiMAX). The model presented in this paper is flexible and can be adopted to the other WLANs.

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