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Perturbation in Population of Pulse-Coupled Oscillators Leads to Emergence of Structure

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Abstract: A new synchronization model based on pulse-coupled oscillators is proposed. A population of coupled oscillators is represented as a cellular automaton. Each cell periodically enters a firing state. Firing of a cell is sensed by other cells in a neighborhood of radius R. As a result the sensing cell may change its firing rate. The interaction strength between a firing and a sensing cell decreases with the squared distance between the two cells. For most starting conditions waves of synchronized firing cells emerge. Simulations indicate that for certain parameter values the emergence of synchronization waves occurs only if there is dispersion in the intrinsic firing frequencies of the cells. Emergence of synchronization waves is an important feature of the model.
Keywords: Cellular Automaton, Synchronization, Emergence, Pulse-Coupled Oscillators

1 Introduction

Many physical and biological systems can be described by mathematical models of coupled oscillators. Examples include earthquake formation [1], synchronously firing neurons [2], synchronous flashing of fireflies [3] and heart pacemaker cells [4].

Pulse-coupled oscillators are a special case when the interaction between oscillators is pulsatile. Each oscillator can enter a firing state when emits a pulse of physical signal. This signal is received by other oscillators in the population and as a result they may change their own oscillation frequency.

A model of pulse-coupled oscillators in the area of cardiology has been first described by Peskin [4]. The model has proved itself to be applicable in many other areas and was extensively studied [5, 6]. Many variations of the model have been considered, including different coupling mechanisms and topolgies.

A population of pulse-coupled oscillators on a 2D grid is considered. Changes in each oscillator occur at discrete time moments. Oscillators (grid cells) interact only if the distance between them is less than a given radius. The interaction strength decreses with the squared distance between the interacting cells.

In many models of pulse-coupled oscillators a total synchronization of the population is eventually observed. However, for the proposed model, extensive simulations indicate that a total synchronization of the population does not occur. Instead, waves of synchronously firing oscillators emerge (see section 3). This is an interesting feature of the model that indicate a good potential for applications.

2 Proposed Oscillator Population Model

2.1 Grid model

A population of pulse-coupled oscillators arranged on a 2-dimensional grid is considered. The oscillator's states are synchronously updated at regular intervals, called generations. The structure of each oscillator consist of a classical integrate-and-fire mechanism combined with a separate mechanism for sensing and integrating the signal emitted by neigbouring oscillators.

A classical integrate-and fire oscillator consists of a unit which integrates an input signal until the output value of the integrator reaches a certain threshold. Then the oscillator enters firing state, the integrator discharges and the process repeats by itself. The time elapsed between two consecutive firing states is called the firing period of the oscillator. The discharge time is usually considered negligable compared to the firing period.

The firing period of the oscillator can be changed in at least two ways: by changing the slope of the integrator or by changing the firing threshold.

In the proposed model the second approach is considered. The integrator has a constant slope (which is a parameter of the oscillator). It generates a signal which increases proportionally with time (the number of generations in our discrete model).

A separate mechanism in each oscillator is responsible for sensing the signals emitted by other oscillators. The energy of the detected signals are integrated by a leaky integrator, i.e. the accumulated energy of the integrator decays exponentially in time.

An oscillator meats the firing condition if the output value of the integrate-and-fire mechanism summed with the weighted value of the sensing integrator is greater than one. This is equivalent to a classic integrate-and-fire oscillator with a non-constant firing threshold: at each moment the firing threshold is one minus the (weighted) value of the sensing integrator.

A particular feature of the model is that the intensity of the signal emitted by a firing oscillator decreases with the squared distance. This models the behaviour of a physical signal (like light or sound) propagating in the three-dimensional space.

The proposed model limits the interaction distance between two cells to a certain value called the interaction radius. This corresponds to a sensitivity threshold of a cell.

2.2 Oscillator state and update rules

The internal state of an oscillator is represented by a set of three variables (P, S, F) with the following meaning :

- P, called potential, represents the current value of the constant-slope integrator;
- S, called signal, represents the current value of the sensing integrator;
- F, called firing state, is a boolean value which is true when the oscillator is in firing state in the given generation.

At each generation the state variables of each oscillator are updated according to the following rules:

The potential P is incremented with a constant value E. The value of E is an oscillator-specific parameter and determines the slope of the integrator.

$$P <= P + E \tag{1}$$

The signal S is calculated by summing the intensities of the signals emitted by all oscillators in the interaction radius which are in firing state, then decrementing S by an amount proportional to the value of S in the previous generation:

$$S <= \sum_k \frac{1}{d_k^2} \tag{2}$$

where d_k is the distance between the oscillator and a neighbouring oscillator k in the interaction domain $d_k <= R$ which is in firing state and D is an oscillator-specific decay parameter.

The firing state F of an oscillator is determined by comparing the weighted summ of the potential P and signal S with a constant threshold:

$F <= true$ **if** $P + KS >= 1$
$F <= false$ **otherwise**

where K is an oscillator-specific coupling parameter.

A cell is in firing state for only one generation. In the next generation returns to normal state and the potential P is reset to zero:

$P <= 0$ **if** $F = true$ in the previous generation

The model implies that if the oscillator has no firing neighbors inside the interaction radius R than it will fire periodically with a firing rate determined by the parameter E.

The potential is incremented in each generation with E and when it becomes grater than 1, the oscillator enters firing state for one generation. On the next generation the potential is reset to zero and the process repeats by itself.

However, if there are firing oscillators in the neigbourhood of the oscillator then the 'radiated energy' of the firing increases the signal value of the oscillator. This value, multiplied by the coupling parameter K is added to the potential and together determine the firing condition of the oscillator. Thus, if many oscillators fire in the neigbourhood of the oscillator, the oscillator will fire more frequently.

3 Experimental Results

The proposed automaton model is simulated for different values of the parameters E, D and K. Different interaction radii R are also considered.

The potential P of each oscillator is initialized to a random value uniformly distributed in the interval [0, 1). The starting value of the signal S is set to zero for each oscillator.

In each experiment the decay and the coupling parameters D and K are set to the same value for each cell.

In some experiments the parameter E which determines the intrinsic firing frequency of the cell is chosen to have the same value. In other experiments the E value is peturbed by a small random noise, so that each cell is initalized with different values of E.

In experiments where the parameter is the same for all cells, the evolution of the cell array displays a random pattern of firing cells. This pattern is maintained along more than 100,000 generations.

However, if the value of the parameter E is perturbed with a small noise, in a few hundred generations clusters of firing cells emerge. Waves of synchronously firing cells emerge tipically after 600-1000 generations.

Figure 1 depicts a typical evolution of the automaton on a grid of 150 x 150 cells.

Parameter values are E = 0.05, D = 0.7, K = 0.1 and R = 30. The initial value of parameter E for each cell is perturbed by a random value in the range [0, 0.005] around the central value of 0.05. In the diagrams of Figure 1 the potential values of the cells are displayed in red color with intensity proportional to the potential. Cells in the firing state are indicated in a bright color.

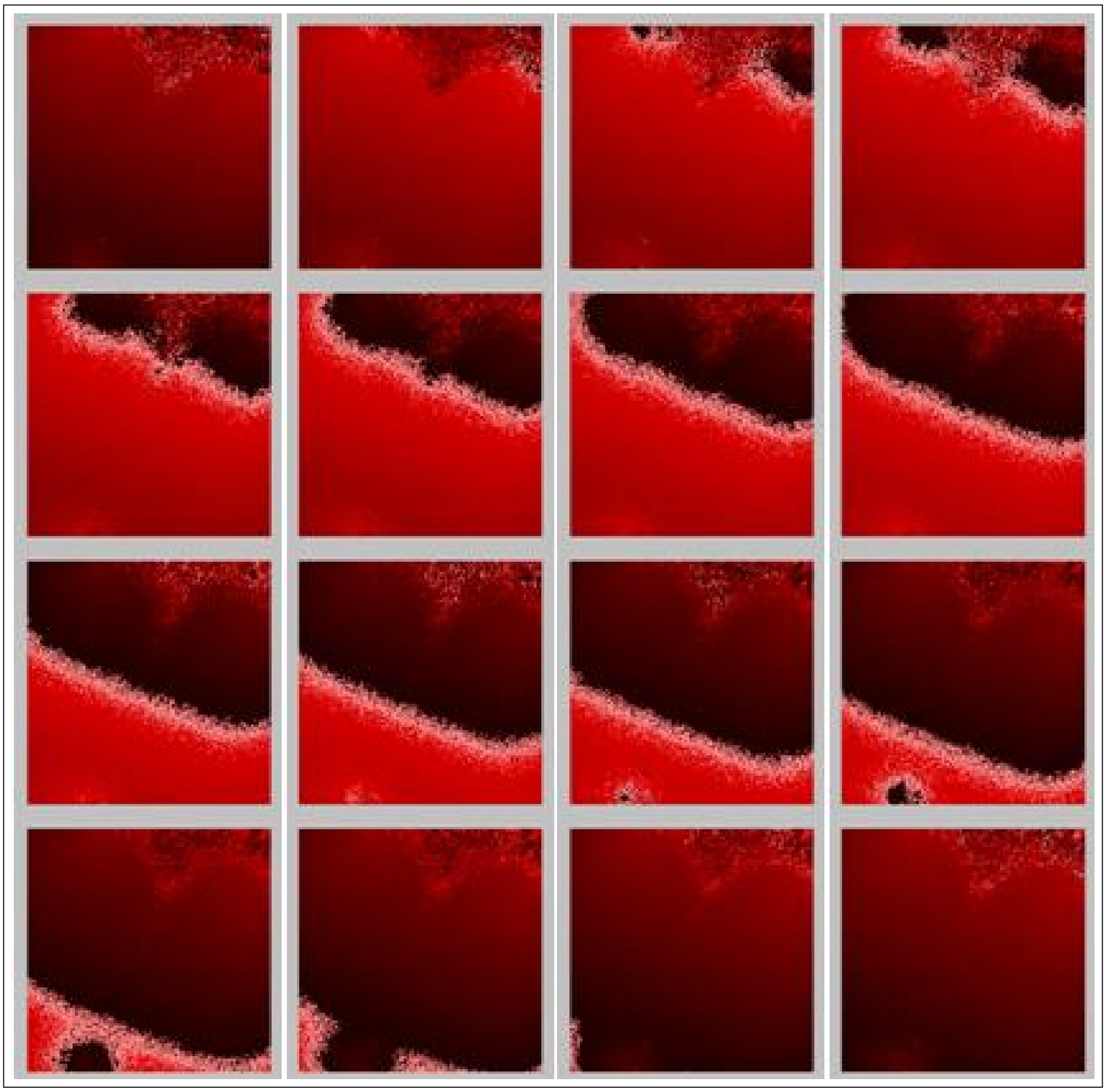

Figure 1: A typical evolution of the oscillator population. Parameter setting: E = 0.05, D = 0.7, K = 0.2, R = 30. The variance of the E parameter was set to 0.005. The sequence of images represents 16 consecutive states of the oscillator array for generations 2400 to 2415. The emergence of a synchronized front of firing oscillators can be observed in this sequence. This front travels through the oscillator array and disappears at the array boundary. After 16 generations the oscillator array returns to a similar state and the process repeats itself.

4 Conclusions

A new model of pulse-coupled oscillators is proposed and investigated. A population of pulse-coupled oscillators on a 2D grid is considered. Changes in each oscillator occur at discrete time intervals. The interaction strength decreases with the squared distance between interacting cells. The interaction distance is limited by a maximum radius. A total synchronization of the population is eventually observed in many models of pulse-coupled oscillators. Simulations indicate that total synchronization of the population does not occur in the propsed model. Instead some synchronization patterns emerge. These emergent patterns are observed as waves of synchronously firing oscillators. The condition of emergence is the existence of a small perturbation in the intrinsic frequencies of individual oscillators. If all oscillators have the same intrinsic frequency then the synchronization patterns do not emerge. The proposed model describes a new auto-organization paradigm conditioned by the presence of a noise mandatory in inducing an emergent synchronization process. Therefore the model provides a way of understanding self-organization in some natural or artificial systems.

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