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# Combination of Evidential Sensor Reports with Distance Function and Belief Entropy in Fault Diagnosis

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**Abstract:** Although evidence theory has been applied in sensor data fusion, it will have unreasonable results when handling highly conflicting sensor reports. To address the issue, an improved fusing method with evidence distance and belief entropy is proposed. Generally, the goal is to obtain the appropriate weights assigning to different reports. Specifically, the distribution difference between two sensor reports is measured by belief entropy. The diversity degree is presented by the combination of evidence distance and the distribution difference. Then, the weight of each sensor report is determined based on the proposed diversity degree. Finally, we can use Dempster combination rule to make the decision. A real application in fault diagnosis and an example show the efficiency of the proposed method. Compared with the existing methods, the method not only has a better performance of convergence, but also less uncertainty.
**Keywords:** Dempster-Shafer evidence theory, sensor data fusion, fault diagnosis, evidence distance, belief entropy, information volume.

## 1 Introduction

In mechanical engineering, some systems are very complex, which might have many components, reflecting with each other [11,38,44,71]. It is likely that something happens unexpectedly in the systems and causes serious problems due to a variety of reasons, such as unfavorable weather, bad environment or a long time of working. As a result, making full use of sensor reports information is extremely significant to make a reasonable decision in fault diagnosis [58,81].

In order to make a rational decision when using sensor data fusion technology, some works have been proposed to handle uncertainty [39, 45, 59, 77], such as fuzzy set theory [15, 63, 66, 74, 76, 83], Z numbers [30, 31], D numbers [8, 9, 41, 64, 65], R numbers [47, 48] and so on. One of the most used math tools in sensor data fusion is evidence theory [4, 49]. This theory can efficiently model uncertain information with basic probability assignment (BPA), or called as belief function [21]. In addition, the Dempster rule can efficiently combine the sensor reports from different sources [79]. Due to the desirable properties, evidence theory has been accepted as de facto standard in decision making [10, 28, 78], risk and reliability analysis [13, 27, 40], system optimization [67] and pattern recognition [23, 24, 34, 46, 57].

However, an open issue of evidential sensor data fusion is that the illogical results will be obtained when sensor reports conflict with each other in a high degree [20, 56, 75]. Many methods were presented to address this issue [60, 80]. For example, Yager [70] removed the process of normalizing in D-S combination rule, Smets [50, 51] proposed the conjunctive and disjunctive rules, Murphy [42] combined the conflicting evidence with average operation, Fan and Zuo [16] presented a combination method to fuse conflicting evidence in fault diagnosis, Dubois and Prades method [12], Lefevre *et al.* [33] and so on. Recently, Jiang *et al.* [25] applied belief entropy into sensor data fusion and received the best performance.

Although these methods have some advantages, one of the disadvantages is that some information is not fully used. For example, the distance information and the difference of information volume are not considered in [25]. In this paper, we proposed an improved evidential method, which is conceptually simple, and yet is able to provide much better accuracy and less uncertainty. The basic idea is to obtain the appropriate weights for different reports. The distribution difference between two BPAs is first measured by belief entropy [6]. Then the diversity degree among BPAs can be obtained by combining distribution difference and evidence distance [29]. According to it, the weight of each BPA can be determined. Finally, we can make a decision for fault diagnosis by using Dempster combination rule. An application in fault diagnosis and an example show that our proposed approach can not only increase the accuracy of fault diagnosis but also decrease the uncertain information volume, which is more reasonable.

The remainder of this paper is organized as follows. Section 2 introduces some backgrounds, including Dempster-Shafer Evidence Theory, Evidence Distance and Belief entropy. Section 3 formulates the proposed method for evidential sensor data fusion. A real application in fault diagnosis and an example are given in Section 4 to show the efficiency of the method. The conclusions are in Section 5.

# 2 Preliminaries

## 2.1 Dempster-Shafer Evidence Theory

The application in data fusion needs efficient math tools [38]. Dempster-Shafer Evidence Theory, proposed by Dempster [4] and Shafer [49], is effective to handle uncertain information.

**Definition 1.** Let $X$ be a set of mutually exclusive and collectively exhaustive events, shown as follow [4, 49]:

$$X = \{\theta_1, \theta_2, \cdots, \theta_i, \cdots, \theta_{|X|}\} \tag{1}$$

where set $X$ is called a frame of discernment, whose power set is:

$$2^X = \{\emptyset, \{\theta_1\}, \cdots, \{\theta_{|X|}\}, \{\theta_1, \theta_2\}, \cdots, \{\theta_1, \theta_2, \cdots, \theta_i\}, \cdots, X\} \tag{2}$$

**Definition 2.** For a frame of discernment $X = \{\theta_1, \theta_2, \cdots, \theta_{|X|}\}$, a mass function is a mapping $m$ from $2^\theta$ to [0,1].

$$m : 2^\theta \to [0, 1] \tag{3}$$

which satisfies the following condition:

$$m(\emptyset) = 0, \sum_{A \in 2^X} m(A) = 1 \tag{4}$$

$A$, a member of the power set, is called a focal element of the mass function, or named as basic probability assignment (BPA).

BPA is the key issue in evidence theory and many relative processing are presented such as negation [19, 73], correlation [26] and divergence measure [17, 52].

**Definition 3.** Given two BPAs, $m_1$ and $m_2$, they can be combined by,

$$m(A) = \begin{cases} 0, & A = \emptyset \\ \frac{1}{1-K} \sum\limits_{B \cap C = A} m_1(B) m_2(C), & A \neq \emptyset \end{cases} \tag{5}$$

with

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C) \tag{6}$$

where $K$ is a parameter that reflects the conflict between $m_1$ and $m_2$. If $K = 0$, $m_1$ and $m_2$ have no contradiction.

If $K = 1$, they are totally conflict. Many open issues about conflict management are still not well addressed [2]. Some alternatives are proposed to modify the combination rule [53,54,61,68], others are presented to modify the data models [80] or handle this problem under open world assumption [55, 56].

## 2.2 Evidence distance

**Definition 4.** Let $m_1$ and $m_2$ be two BPAs on the same frame of discernment of $X$, which contains $N$ mutually exclusive and exhaustive hypotheses. The distance between $m_1$ and $m_2$ is [29]:

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}(\overrightarrow{m_1} - \overrightarrow{m_2})^T D (\overrightarrow{m_1} - \overrightarrow{m_2})} \tag{7}$$

where D is a $2^N \times 2^N$ matrix whose elements are

$$D(A, B) = \frac{|A \cap B|}{|A \cup B|}, \quad A, B \in P(X).$$

## 2.3 Belief entropy

It should be pointed out that uncertainty measurement, decision making and optimization under uncertainty is still an open issue [14,22,37]. Entropy is an efficient tool to model uncertainty [7, 32, 72]. Recently, a new belief entropy, named as Deng entropy was proposed [6]. It has a good performance in measuring uncertainty. Also, it has a backward compatibility, which means when the uncertain information is represented by probability distribution, belief entropy will degenerate to Shannon entropy [1, 3, 35, 43].

**Definition 5.** Let $A$ be a proposition of BPA, $|A|$ is the cardinality of $A$. Then, belief entropy is defined as [6]:

$$E_d = -\sum_{A \subseteq X} m(A) log \frac{m(A)}{2^{|A|} - 1} \tag{8}$$

When the BPA has only one element, which means $|A| = 1$, then it can be written as Shannon entropy [6].

$$E_d = -\sum_{A \subseteq X} m(A) log \frac{m(A)}{2^{|A|} - 1} = -\sum_{A \subseteq X} m(A) log\ m(A) \tag{9}$$

## 3 Evidential sensor data fusion

This section formulates a new weighted average approach for evidential sensor data fusion. The proposed method considered both evidence distance and belief entropy to obtain the appropriate weights assigning to different data reports. Using the weight to pre-treat the multi-source reports, and making the final decision by Dempster combination rule. The flow chart is shown in Figure 1.

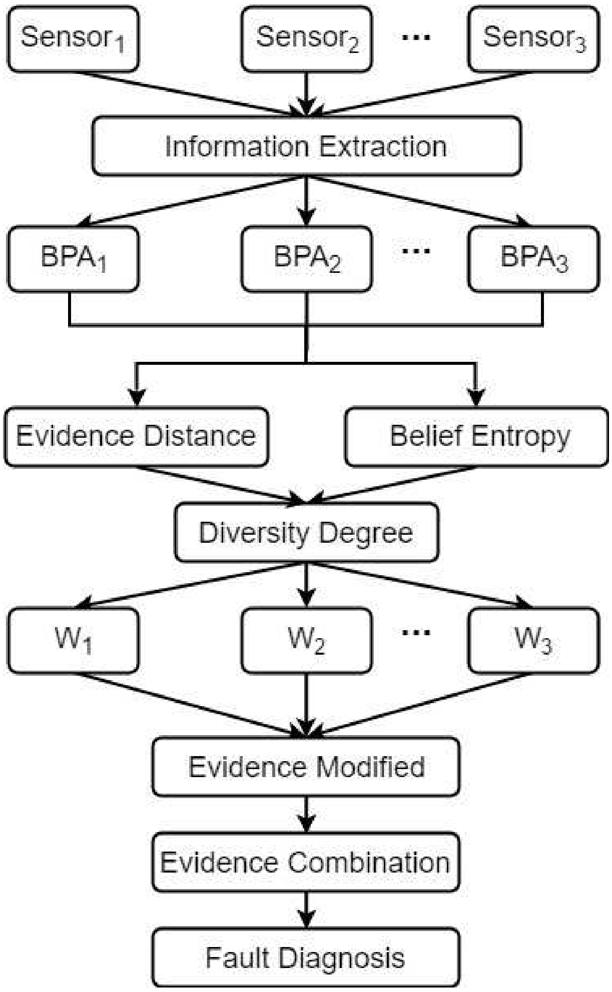


Figure 1: Structure of the proposed sensor data fusing method

**Definition 6.** Let $E_{di}, E_{dj}$ be the belief entropy of $m_1, m_2$, then the distribution difference is defined:

$$\alpha_{ij} = e^{|E_{di} - E_{dj}|} \tag{10}$$

**Definition 7.** Let $d_{BPA}(m_i, m_j)$ be the evidence distance of $m_1, m_2$, the diversity degree between two BPAs is defined:

$$D_{ij} = \frac{1}{\alpha_{ij} \times d_{BPA}(m_i, m_j)} \tag{11}$$

**Definition 8.** A diversity of distribution and distance matrix ($DDD$) is also defined:

$$DDD = \begin{pmatrix} 1 & D_{12} & \cdots & D_{1i} & \cdots & D_{1n} \\ D_{21} & 1 & \cdots & D_{2i} & \cdots & D_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ D_{i1} & D_{i2} & \cdots & 1 & \cdots & D_{in} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ D_{n1} & D_{n2} & \cdots & D_{ni} & \cdots & 1 \end{pmatrix} \tag{12}$$

The computation step is shown as following.

- Step 1, collect sensor information and transform into BPAs.
- Step 2, use Equation (7) to calculate the evidence distance between two BPAs, which shows the difference.
- Step 3, use Equation (8) to calculate the belief entropy of BPA, which shows the distributive characteristic.
- Step 4, calculate the diversity degree between two BPAs.
- Step 5, the support of the BPAs is given as:

$$Sup(m_i) = \sum_{j=1,\ j \neq i}^{n} D_{ij} \tag{13}$$

- Step 6, the credibility degree of the BPAs is obtained.

$$Crd_i = \frac{Sup(m_i)}{\sum\limits_{i=1}^{n} Sup(m_i)} \tag{14}$$

- Step 7, it is easy to see that $\sum_{i=1}^{n} Crd_i = 1$. As a result, $Crd_i$ can be the weight of each BPA. $(\omega_1, \omega_2, \cdots, \omega_n) = (Crd_1, Crd_2, \cdots, Crd_n)$. Thus, a new weighted evidence can be obtained, which is:

$$m(a) = \omega_1 \times m_1(a) + \omega_2 \times m_2(a) + \cdots + \omega_n \times m_n(a) \tag{15}$$

- Step 8, use Dempster combination rule to combine the new weighted evidence to get the result. Furthermore, if the number of original evidence is $n$, then the new evidence should be combined for $(n-1)$ times [25].

It should be noticed that if $d_{BPA}(m_i, m_j) = 0$, which means that there is no conflict between $m_i$ and $m_j$. In this situation, the value of diversity degree $D_{ij}$ cannot be determined. So we proposed our own solution, which can be discussed further.

First, if there are only three BPAs $m_1, m_2, m_3$ and $m_1 = m_2$, the two BPAs can be regarded as the same one, then use Murphy's method, which is assigning the weight equally to each BPA.

Second, if there are more than three evidences. Supposing there are $n$ BPAs, which can be divided into $m$ groups according to their $d_{BPA}$. If there are only two groups, it will just be the same as the first situation. If there are more than two groups, and each group has $k$ BPAs, noted as $k_1, k_2, \cdots, k_m$, and $\sum_{i=1}^{m} k_i = n$.

$$\begin{array}{cccc} \{m_{11} & m_{12} & \cdots & m_{1k_1}\} \\ \{m_{21} & m_{22} & \cdots & m_{2k_2}\} \\ \vdots & \vdots & \vdots & \vdots \\ \{m_{m1} & m_{12} & \cdots & m_{mk_m}\} \end{array} \tag{16}$$

Then, select one BPA from each group. Suppose they are $m_{11}, m_{21}, \cdots, m_{m1}$, and use the proposed method to obtain the weight, which is

$$\{w(m_{11}), w(m_{21}), \cdots, w(m_{m1})\} = \{\omega_1, \omega_2, \cdots, \omega_m\} \tag{17}$$

Since $w = \sum_{i=1}^{m} k_i \omega_i$, and the finally weight can be gained.

$$\begin{aligned} &\{\omega_{11}, \omega_{12}, \cdots, \omega_{1k_1}, \omega_{21}, \omega_{22}, \cdots, \omega_{2k_2}, \cdots, \omega_{m1}, \omega_{m2}, \cdots, \omega_{mk_m}\} \\ &= \{\underbrace{\frac{\omega_1}{w}, \frac{\omega_1}{w}, \cdots, \frac{\omega_1}{w}}_{k_1}, \underbrace{\frac{\omega_2}{w}, \frac{\omega_2}{w}, \cdots, \frac{\omega_2}{w}}_{k_2}, \cdots, \underbrace{\frac{\omega_m}{w}, \frac{\omega_m}{w}, \cdots, \frac{\omega_m}{w}}_{k_m}\} \end{aligned} \tag{18}$$

Two examples are given to illustrate how it works.

**Example 1.** Suppose the frame of discernment is $X = \{a_1,\ a_2,\ a_3\}$, and there are three BPAs.

$$m_1(a_1) = 0.3,\ m_1(a_2, a_3) = 0.7$$

$$m_2(a_1) = 0.3,\ m_2(a_2, a_3) = 0.7$$

$$m_3(a_1) = 0.8,\ m_3(a_2, a_3) = 0.2$$

Then the weight should be obtained as $W(1/4, 1/4, 1/2)$.

**Example 2.** Suppose the frame of discernment is also $X = \{a_1,\ a_2,\ a_3\}$, and there are four BPAs.

$$m_1(a_1) = 0.3,\ m_1(a_2, a_3) = 0.7,$$

$$m_2(a_1) = 0.3,\ m_2(a_2, a_3) = 0.7,$$

$$m_3(a_1) = 0.8,\ m_3(a_2, a_3) = 0.2.$$

$$m_4(a_1) = 0.6,\ m_4(a_2, a_3) = 0.4.$$

In this example, we can just consider $m_2, m_3$ and $m_4$, then use the proposed method to gain the weights, which are

$$\omega_2 = 0.2585, \omega_3 = 0.3072, \omega_4 = 0.4343$$

Finally, re-assign the weights to get the result: $W = (0.2054, 0.2054, 0.2441, 0.3451)$.

# 4 Experiments

## 4.1 Application in fault diagnosis

The complex systems is very complicated since each factor in the system interacting with each other in a very complicated way. To address this issue, network analysis [18,36,62,69,82] and data fusion based technology are presented to deal with complexity and guarantee the reliability of the complex system. [25] gave a case of motor rotor fault diagnosis, where the vibration signal is collected by acceleration sensor ($m_1$), velocity sensor ($m_2$), and displacement sensor ($m_3$). The possible faults including normal operation ($F_1$), unbalance ($F_2$), misalignment ($F_3$), pedestal looseness ($F_4$). The collected data report is shown in Table 1, where $X$ is a frame of discernment, and $X = \{F_1, F_2, F_3, F_4\}$.

Table 1: Output of the multi-senors [25]

| $m_i$ | $F_1$ | $F_2$ | $F_3$ | $F_4$ | X |
|---|---|---|---|---|---|
| $m_1$ | 0.06 | 0.68 | 0.02 | 0.04 | 0.20 |
| $m_2$ | 0.02 | 0 | 0.79 | 0.05 | 0.14 |
| $m_3$ | 0.02 | 0.58 | 0.16 | 0.04 | 0.20 |

The computation steps are as following.
By Equation (7), the evidence distance between two BPAs,

$$d_{BPA}(m_1, m_2) = 0.7276, \quad d_{BPA}(m_1, m_3) = 0.1249, \quad d_{BPA}(m_2, m_3) = 0.6063$$

Using Equation (8) to get the belief entropy of each BPA.

$$E_{d1} = 2.1663, \quad E_{d2} = 1.5417, \quad E_{d3} = 2.4232$$

The distribution difference is obtained by Equation (10).

$$\alpha_{12} = 1.8674, \quad \alpha_{13} = 1.2930, \quad \alpha_{23} = 2.4145$$

And the diversity degree calculated by Equation (11) are:

$$D(m_1, m_2) = 0.7360, \quad D(m_1, m_3) = 6.1923, \quad D(m_2, m_3) = 0.6831$$

Finally, the weights by Equation (14):

$$\omega_1 = 0.4551, \quad \omega_2 = 0.0932, \quad \omega_3 = 0.4517$$

According to the weights, a new set of data can be got by calculating:

$$m(F_1) = 0.06 \times 0.4551 + 0.02 \times 0.0932 + 0.02 \times 0.4517 = 0.0382$$

In the same way, we can calculate $m(F_2) = 0.5714, m(F_3) = 0.1550, m(F_4) = 0.0409, m(X) = 0.1944$.

Therefore, the decision can be made by fusing $\{0.0382, 0.5714, 0.1550, 0.0409, 0.1944\}$ with Dempster combination rule for 2 times, which are shown in Table 2. The results of other methods are also listed as a comparison. Following [25], we will use $\Delta = 0.7$ as the threshold.

Since the fault $F_2$ has a belief degree of 89.18%, it can be told that unbalance is the fault of the equipment. And compared with other methods, the new method performs much better. The belief entropy in Figure 2 is the smallest, which means that the proposed method has the smallest uncertain information volume.

Table 2: Comparison of the result of several existing methods

| Combination rule | $F_1$ | $F_2$ | $F_3$ | $F_4$ | X | Diagnosis result |
|---|---|---|---|---|---|---|
| Dempster [4, 49] | 0.0205 | 0.5229 | 0.3933 | 0.0309 | 0.0324 | Uncertainty |
| Murphy [42] | 0.0112 | 0.6059 | 0.3508 | 0.0153 | 0.0168 | Uncertainty |
| Jiang [25] | 0.0111 | 0.7265 | 0.2313 | 0.0144 | 0.0168 | Unbalance |
| Deng [5] | 0.0111 | 0.7728 | 0.1851 | 0.0139 | 0.0165 | Unbalance |
| Proposed method | 0.0106 | 0.8918 | 0.0713 | 0.0115 | 0.0148 | Unbalance |

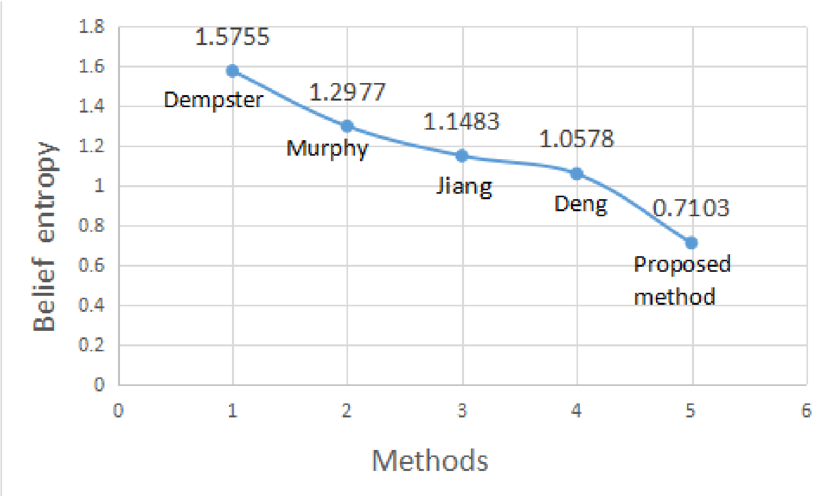


Figure 2: Belief entropy comparison.

### 4.2 Example

The proposed method is not only efficient in dealing with uncertainty, but also be false-evidence resilient. Suppose the system has collected five evidences from five different sensors [5], which are shown as follows:

$$m_1 : m_1(A) = 0.5,\ m_1(B) = 0.2,\ m_1(C) = 0.3;$$
$$m_2 : m_2(A) = 0,\ m_2(B) = 0.9,\ m_2(C) = 0.1;$$
$$m_3 : m_3(A) = 0.55,\ m_3(B) = 0.1,\ m_3(C) = 0.35;$$
$$m_4 : m_4(A) = 0.55,\ m_4(B) = 0.1,\ m_4(C) = 0.35;$$
$$m_5 : m_5(A) = 0.55,\ m_5(B) = 0.1,\ m_5(C) = 0.35;$$

The results are shown in Table 3, Obviously, the second evidence $m_2$ is conflicting with the others. Although there are more and more evidences that support $A$, Dempster's method cannot reach a reasonable result. $m(A)$ will always be zero as long as one evidence does not support it. While other methods revise the disadvantage. Apparently, the proposed method can have a probability 0.7663 to support $A$ when the third evidence is fused, and it always has the highest support.

In previous work, some information is not fully used. In our proposed method, not only the distance information between BPA, but also the difference of information volume of each BPA is considered. As a result, our method can efficiently measure the support degree of each sensor report. That is, if one report is more reliable, more weights will be assigned to this report. Therefore, the result of our improved method is more reasonable and more desirable.

Despite the advantages, the proposed method might not be very suitable in some situations.

For example, the calculation process will be extremely complex if the data of evidence is huge. [25] considered belief entropy, and considered evidence distance, while our method considered both of them. Therefore, the calculation complexity might be twice as those methods. In addition, the proposed method might reach an unreasonable result if a bad evidence repeats many times. This is because the bad evidence cannot be identified, and they could have a mutual confirmation, leading to an illogical fusing result.

Table 3: Results of different method

| Combination rule | $m_1,\ m_2$ | $m_1,\ m_2,\ m_3$ | $m_1,\ m_2,\ m_3,\ m_4$ | $m_1,\ m_2,\ m_3,\ m_4,\ m_5$ |
|---|---|---|---|---|
| Dempster [4,49] | m(A)=0 | m(A)=0 | m(A)=0 | m(A)=0 |
| | m(B)=0.8571 | m(B)=0.6316 | m(B)=0.3288 | m(B)=0.1228 |
| | m(C)=0.1429 | m(C)=0.3684 | m(C)=0.6712 | m(C)=0.8772 |
| Murphy [42] | m(A)=0.1543 | m(A)=0.3500 | m(A)=0.6027 | m(A)=0.7958 |
| | m(B)=0.7469 | m(B)=0.5524 | m(B)=0.2627 | m(B)=0.0932 |
| | m(C)=0.0988 | m(C)=0.1276 | m(C)=0.1346 | m(C)=0.1110 |
| Jiang [25] | m(A)=0.4206 | m(A)=0.6819 | m(A)=0.8244 | m(A)=0.8945 |
| | m(B)=0.3944 | m(B)=0.1310 | m(B)=0.0326 | m(B)=0.0071 |
| | m(C)=0.1850 | m(C)=0.1871 | m(C)=0.1430 | m(C)=0.0984 |
| Deng [5] | m(A)=0.1543 | m(A)=0.5816 | m(A)=0.8061 | m(A)=0.8909 |
| | m(B)=0.7469 | m(B)=0.2439 | m(B)=0.0481 | m(B)=0.0086 |
| | m(C)=0.0988 | m(C)=0.1745 | m(C)=0.1457 | m(C)=0.1005 |
| Proposed method | m(A)=0.1543 | m(A)=0.7663 | m(A)=0.8554 | m(A)=0.9063 |
| | m(B)=0.7469 | m(B)=0.0424 | m(B)=0.0082 | m(B)=0.0015 |
| | m(C)=0.0988 | m(C)=0.1913 | m(C)=0.1364 | m(C)=0.0922 |

# 5 Conclusions

In fault diagnosis and other sensor data fusion systems, the reports of different sensors may be influenced by some complex environments, leading them less reliable. Therefore, how to efficiently determine the reliability of each report, or to say, the weight of each report is very important. To address this issue, we propose an improved method based on the belief entropy and the evidence distance. The method considers both the degree of conflict and the difference of information volume among evidences. An application and an example illustrate the efficiency of the method in evidential sensor fusion. It shows that the proposed method is more efficient for highly conflicting evidences with better performance of convergence and less uncertainty, compared with the existing methods.

Some related advantages and disadvantages of different methods are discussed as follows.

- Dempster's method, which can well deal with imprecise and uncertain information, is widely used in fusing information. However, when it comes to highly conflicting evidences, the method will always lead to some illogical results.

- Murphy's method, which uses an average operation to combine the conflicting evidence, is able to deal with highly conflicting evidences to some extent. However, the difference and relationship of evidences is neglected.

- In Jiang's method, belief entropy is used to calculate the weight of each evidence. It considers the difference of evidence, which makes it more reasonable than Murphy's method.

- In Deng's method, which is different from Jiang's method, evidence distance is used to calculate the weight rather than the belief entropy. And the similarity between two evidences is proposed.
- The proposed method, considering both belief entropy and evidence distance, has a better result. Although it might be not very suitable in some situations, it leverages the advantages of Jiang's method and Deng's method.

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## Author contributions. Conflict of interest

The authors contributed equally to this work. The authors declare no conflict of interest.

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