

Information Volume of Mass Function

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Abstract

Given a probability distribution, its corresponding information volume is Shannon entropy. However, how to determine the information volume of a given mass function is still an open issue. Based on Deng entropy, the information volume of mass function is presented in this paper. Given a mass function, the corresponding information volume is larger than its uncertainty measured by Deng entropy. In addition, when the cardinal of the frame of discernment is identical, both the total uncertainty case and the BPA distribution of the maximum Deng entropy have the same information volume. Some numerical examples are illustrated to show the efficiency of the proposed information volume of mass function.

Keywords: information volume, mass function, Shannon entropy, Deng entropy.

1 Introduction

In the past decades, plenty of theories have been developed for expressing and dealing with the uncertainty in the uncertain environment, for instance, the extended probability theory [26], probabilistic linguistic [11, 28], fuzzy linguistic [21, 22], fuzzy logic [10], fuzzy set theory [67], intuitionistic fuzzy sets [20, 36, 48], soft sets [1, 12], Dempster-Shafer evidence theory [6, 46], rough sets [15, 44], Z numbers [23, 25, 35, 59] and D numbers [7, 30, 33]. Because these theories can well handle uncertainty in many kinds of situations, they have been widely applied in various fields, including decision making [13, 19, 40, 69], reliability analysis [27, 66], medical diagnosis [2, 14], multi-source information fusion [29, 55], causal analysis [32], vehicle system [5], and fault detecting [45].

Entropy function is very important in uncertainty modelling [9]. Since firstly derived from thermodynamics, different kinds of entropy have been proposed, such as Shannon entropy [47], Tsallis entropy [50], nonadditive entropy [51], interval valued entropies [61], and fuzzy entropy [3, 4]. Recently, a new entropy, called Deng entropy [8], is presented for measuring the uncertainty in evidence theory. Deng entropy is the generalization of Shannon entropy. Compared with traditional methods, Deng entropy is more reasonable, and it takes both discord and non-specificity into account [49].

Given a probability distribution, its corresponding information volume can be measured by Shannon entropy. However, how to determine the information volume of mass function in evidence theory is still an open issue. In this paper, an information volume of mass function based on Deng entropy is presented. The information volume of mass function is constructed based on the BPA distribution of the maximum Deng entropy. If the mass function is degenerated into probability distribution, the proposed information volume is the same as Shannon entropy. In addition, when the cardinal of the frame of discernment is constant, both the total uncertainty case and the Deng distribution have the same information volume.

The rest of this paper is organized as follows. In section 2, some preliminaries are briefly reviewed. In section 3, based on Deng entropy, the information volume of mass function is proposed. In section 4, numerical examples are expounded to illustrate the proposed definition. In section 5, we have a brief conclusion.

2 Preliminaries

Several preliminaries are briefly introduced in this section, including Dempster-Shafer evidence theory, mass function, Shannon entropy, Deng entropy and the maximum Deng entropy.

2.1 Dempster-Shafer evidence theory

Uncertainty modelling is still an open issue [9, 52]. Dempster-Shafer evidence theory [6, 46] can be used to deal with uncertainty. Besides, evidence theory satisfies the weaker conditions than the probability theory, which provides it with the ability to express uncertain information directly [39]. Therefore, evidence theory has well studied, including evidence reasoning [38, 64, 65, 68], belief rule [57, 58], complex mass functions [53, 54], and generalized Dempster-Shafer structures [60, 62], and it was applied in many areas, such as risk analysis [42, 43], classification [37], data fusion [56], and heuristic representation learning [16]. Some basic conceptions of evidence theory are given as follows:

Definition 2.1: *Frame of discernment and its power set*

Let Θ , called the frame of discernment, denote an exhaustive nonempty set of hypotheses, where the elements are mutually exclusive. Let the set Θ have N elements, which can be expressed as:

$$\Theta = \{\theta_1, \theta_2, \theta_3, \dots, \theta_N\} \tag{1}$$

The power set of Θ , denoted as 2^Θ , contains all possible subsets of Θ and has 2^N elements, and 2^Θ is represented by

$$\begin{aligned} 2^\Theta &= \{A_1, A_2, A_3, \dots, A_{2^N}\} \\ &= \{\emptyset, \{\theta_1\}, \{\theta_2\}, \dots, \{\theta_N\}, \{\theta_1, \theta_2\}, \\ &\quad \{\theta_1, \theta_3\}, \dots, \{\theta_1, \theta_N\}, \dots, \Theta\} \end{aligned} \tag{2}$$

where the element A_k is called the focal element of Θ , if A_k is nonempty.

Definition 2.2: *Mass function*

A mass function is also called Basic probability assignment (BPA), which map m from 2^Θ to $[0, 1]$, and it is defined as follows:

$$m : 2^\Theta \rightarrow [0, 1] \tag{3}$$

which is constrained by the following conditions:

$$\sum_{A \in 2^\Theta} m(A) = 1 \tag{4}$$

$$m(\emptyset) = 0 \tag{5}$$

Compared with probability distribution in probability theory, mass function has more efficient manner to handle uncertainty [63]. In addition, some other extension of mass function in quantum information has been paid attention recently [17].

2.2 Shannon entropy

Entropy plays an important role in measure the uncertainty [34, 41]. In the field of classical probability theory, Shannon entropy [47] is often used to measure the uncertainty of a probability distribution. Consider a probability distribution P defined on the set $\Theta = \{H_1, H_2, H_3, \dots, H_N\}$.

Definition 2.5: *Shannon entropy*

Shannon entropy $H_s(P)$ is defined as follows:

$$H_s(P) = \sum_{\theta \in \Theta} P(\theta) \log\left(\frac{1}{P(\theta)}\right). \tag{6}$$

where $\sum_{\theta \in \Theta} P(\theta) = 1$ and $P(\theta) \in [0, 1]$.

Usually, the base of logarithm is 2, and entropy has the unit of bit. It's not hard to find that $H_s(P)$ is on the scale $[0, \log N]$.

2.3 Deng entropy

In information theory, entropy can be used to measure the uncertainty of a system. Recently, a novel entropy, named as Deng entropy [8], is proposed to measure the uncertainty in evidence theory.

Definition 2.5: *Deng entropy*

Deng entropy is defined as:

$$H_{DE}(m) = - \sum_{A \in 2^\Theta} m(A) \log\left(\frac{m(A)}{2^{|A|} - 1}\right) \tag{7}$$

where $|A|$ is the cardinal of a certain focal element A .

Deng entropy is the generalization of Shannon entropy. When every focal element is singleton, Deng entropy degenerates into Shannon entropy [18].

Through a simple transformation, Eq.(7) can be rewritten as follows:

$$H_{DE}(m) = \sum_{A \in 2^\Theta} \log(2^{|A|} - 1) - \sum_{A \in 2^\Theta} m(A) \log m(A) \tag{8}$$

where $\sum_{A \in 2^\Theta} \log(2^{|A|} - 1)$ and $-\sum_{A \in 2^\Theta} m(A) \log m(A)$ are measurements of nonspecificity and discord, respectively. As a result, Deng entropy is a composite measurement of nonspecificity and discord, which means that it is a tool for measuring total uncertainty [31].

2.4 The maximum Deng entropy

Assume A is the focal element of a certain frame of discernment Θ and $m(A)$ is the BPA for A . According to [24], the analytic solution of the maximum Deng entropy is as follows:

Theorem 2.1: *The analytic solution of the maximum Deng Entropy*

If and only if $m(A) = \frac{(2^{|A|}-1)}{\sum_{A \in 2^\Theta} (2^{|A|}-1)}$, Deng entropy reaches its maximum value. The analytic solution of the maximum Deng entropy is

$$H_{MDE}(m) = \log \sum_{A \in 2^\Theta} (2^{|A|} - 1) \tag{9}$$

3 Information volume of mass function

Given a probability distribution, the associated information volume can be measured by Shannon entropy. However, how to measure the information volume of a given mass function is still an open issue.

In this section, based on Deng entropy, the information volume of mass function is defined.

Definition 3.1: *Information volume of mass function*

Let the frame of discernment be $\Theta = \{\theta_1, \theta_2, \theta_3, \dots, \theta_N\}$. Use index i to denote the times of this loop, and use $m(A_i)$ to denote different mass function of different loops. Based on Deng entropy, the information volume of mass function can be calculated by following steps:

step 1: Input mass function $m(A_0)$.

step 2: Continuously separate the mass function of the element whose cardinal is larger than 1 until convergence. Concretely, repeat the loop from step 2-1 to step 2-3 until Deng entropy is convergent.

step 2-1: Focus on the element whose cardinal is larger than 1, namely, $|A_i| > 1$. And then, separate its mass function based on the proportion of the BPA of the maximum Deng entropy:

$$m(A_i) = \frac{(2^{|A_i|} - 1)}{\sum_{A_i \in 2^\Theta} (2^{|A_i|} - 1)} \tag{10}$$

For example, given a focal element $A_{i-1} = \{\theta_x, \theta_y\}$ and its mass function $m(A_{i-1})$, the separating proportion is that $\frac{1}{5} : \frac{1}{5} : \frac{3}{5}$. The i th times of separation divides $m(A_{i-1})$ and yields following new mass function: $m(X_i)$, $m(Y_i)$, $m(Z_i)$, where $X_i = \{\theta_x\}$, $Y_i = \{\theta_y\}$ and $Z_i = \{\theta_x, \theta_y\}$. In addition, they satisfy these equations:

$$m(X_i) + m(Y_i) + m(Z_i) = m(A_{i-1}) \tag{11}$$

$$m(X_i) : m(Y_i) : m(Z_i) = \frac{1}{5} : \frac{1}{5} : \frac{3}{5} \tag{12}$$

step 2-2: Based on Deng entropy, calculate the uncertainty of all the mass functions except for those who have been divided. The result is denoted as $H_i(m)$.

step 2-3: Calculate $\Delta_i = H_i(m) - H_{i-1}(m)$. When Δ_i satisfies following condition, jump out of this loop.

$$\Delta_i = H_i(m) - H_{i-1}(m) < \varepsilon \tag{13}$$

where ε is the allowable error.

step 3: Output $H_{IV-mass}(m) = H_i(m)$, which is the information volume of the mass function.

4 Numerical examples and discussions

In this section, some examples are expounded to better understand the definition for the proposed information volume of mass function, and the discussion is followed after every example. In the following examples, the base of the logarithmic function is 2, and the allowable error is 0.001.

Example 4.1:

Consider the focal element be $X = \{\theta_1\}$, $Y = \{\theta_2\}$ and $Z = \{\theta_3\}$. Let the mass function be $m_0(X) = m_0(Y) = m_0(Z) = \frac{1}{3}$.

Because there is no focal element whose cardinal is larger than 1, the step 2-1 can be skipped for all the times of the loop. Then, in step 2-2, use Deng entropy to calculate the uncertainty of this mass function:

$$H_i(m) = -\frac{1}{3} \log_2\left(\frac{1}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) = 1.584963 \tag{14}$$

After going through the loop again, the new $H_i(m)$ is also 1.585 since step 2-1 is always skipped. As a result, we escape from the loop and get the information volume of this mass function $H_{IV-mass}(m) = 1.584963$.

Actually, this form of mass function is the probability distribution $P_1 = P_2 = P_3 = \frac{1}{3}$. Hence, when the mass function degenerates into the probability distribution, the value of $H_{IV-mass}(m)$ is identical to the Shannon entropy.

Example 4.2:

Consider the frame of discernment be $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Let the mass function be $m_0(\{\theta_1\}) = m_0(\{\theta_2\}) = m_0(\{\theta_3\}) = m_0(\{\theta_1, \theta_2\}) = m_0(\{\theta_1, \theta_3\}) = m_0(\{\theta_2, \theta_3\}) = m_0(\{\theta_1, \theta_2, \theta_3\}) = \frac{1}{7}$.

The information volume of this mass function can be calculated by **Definition 3.1**. The convergence procedure of $H_i(m)$ is listed in Table 1.

Table 1: The convergence procedure of $H_i(m)$

i	$H_i(m)$	i	$H_i(m)$
1	3.887675	9	5.178227
2	4.409314	10	5.187146
3	4.724509	11	5.192498
4	4.914440	12	5.195709
5	5.028700	13	5.197636
6	5.097366	14	5.198792
7	5.138606	15	5.199486
8	5.163366		

According to Table 1, when $i = 15$, $H_i(m) - H_{i-1}(m) < 0.001$, which means that $H_i(m)$ finally converges to 5.199486. Hence the information volume of this mass function is $H_{IV-mass}(m) = 5.199486$.

If we use Deng entropy to measure the uncertainty of this mass function, the result is as follows:

$$\begin{aligned}
 H_{DE}(m) &= -\frac{1}{7} \log_2\left(\frac{\frac{1}{7}}{2^1 - 1}\right) \times 3 - \frac{1}{7} \log_2\left(\frac{\frac{1}{7}}{2^2 - 1}\right) \times 3 - \frac{1}{7} \log_2\left(\frac{\frac{1}{7}}{2^3 - 1}\right) \\
 &= 3.887675
 \end{aligned}
 \tag{15}$$

Compared $H_{IV-mass}(m)$ with $H_{DE}(m)$ in this example, $H_{IV-mass}(m)$ is larger than $H_{DE}(m)$, which shows that, given a mass function, the corresponding information volume is larger than its uncertainty measured by Deng entropy.

Example 4.3:

Consider the frame of discernment be $U = \{\theta_1, \theta_2\}$, $X = \{\theta_1\}$ and $Y = \{\theta_2\}$ be singletons. Let the mass function be $m_0(X) = m_0(Y) = \frac{1}{5}$ and $m_0(U) = \frac{3}{5}$, which is the BPA distribution of the maximum Deng entropy when the cardinal of the frame of discernment is 2.

The information volume of this mass function can be calculated by **Definition 3.1**, whose calculating procedure is illustrated in Figure 1. For the convenience of comprehension, the calculating procedure can be abstracted as a directed acyclic graphical model shown in Figure 2.

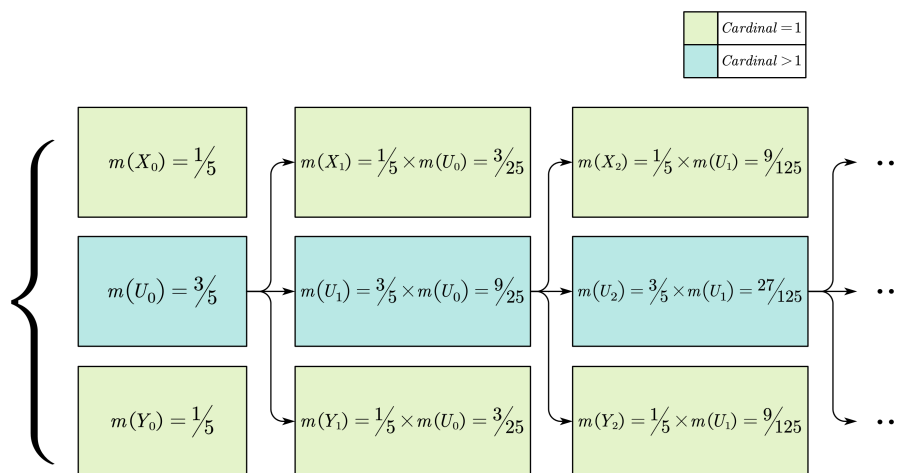


Figure 1: The calculating procedure of **Example 4.3**

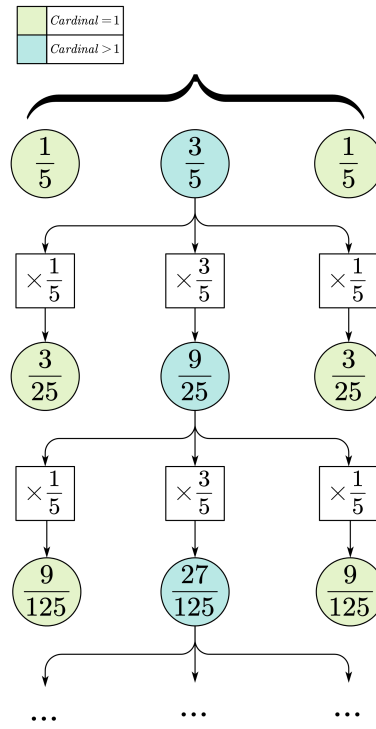


Figure 2: The directed acyclic graphical model of **Example 4.3**

Table 2: The convergence procedure of $H_i(m)$

i	$H_i(m)$	i	$H_i(m)$
1	2.321928	8	3.396431
2	2.764107	9	3.408809
3	3.029415	10	3.416236
4	3.188600	11	3.420692
5	3.284110	12	3.423366
6	3.341417	13	3.424970
7	3.375801	14	3.425933

Then, the convergence procedure of $H_i(m)$ is listed in Table 2.

According to Table 2, when we continuously separate the BPA of the element whose cardinal is larger than 1, the Δ_i of Deng entropy becomes smaller and smaller. When $i = 14$, $H_i(m) - H_{i-1}(m) < 0.001$, which means that $H_i(m)$ finally converges to 3.425933.

Hence, when the cardinal of the frame of discernment is 2, the information volume of the BPA distribution of the maximum Deng entropy is $H_{IV-mass}(m) = 3.425933$.

Example 4.4:

Consider the frame of discernment be $\Theta = \{\theta_1, \theta_2\}$. Let the mass function be $m_0(\Theta) = m_0(\{\theta_1, \theta_2\}) = 1$, which is called the total uncertainty case when the cardinal of the frame of discernment is 2.

The information volume of this total uncertainty case can be calculated by **Definition 3.1**. The calculating procedure can be abstracted as a directed acyclic graphical model shown in Figure 3, and the convergence procedure of $H_i(m)$ is listed in Table 3.

According to Table 3, when $i = 15$, $H_i(m) - H_{i-1}(m) < 0.001$, which means that $H_i(m)$ finally converges to 3.425933. Hence, when the cardinal of the frame of discernment is 2, the information volume of the total uncertainty case is $H_{IV-mass}(m) = 3.425933$.

Compared **Example 4.3** with **Example 4.4**, we can find that, although the directed acyclic graphical models of each example is not the same, the $H_{IV-mass}(m)$ of them is identical.

It can be concluded that, when the cardinal of the frame of discernment is identical, the BPA

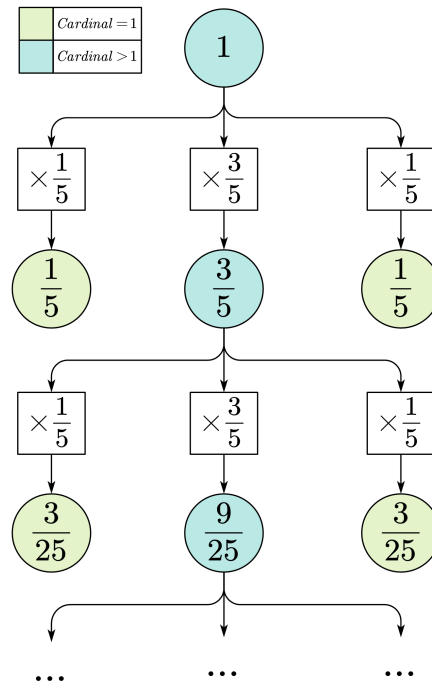


Figure 3: The directed acyclic graphical model of **Example 4.4**

Table 3: The convergence procedure of $H_i(m)$

i	$H_i(m)$	i	$H_i(m)$
1	1.584963	9	3.396431
2	2.321928	10	3.408809
3	2.764107	11	3.416236
4	3.029415	12	3.420692
5	3.188600	13	3.423366
6	3.284110	14	3.424970
7	3.341417	15	3.425933
8	3.375801		

distribution of the maximum Deng entropy and the total uncertainty case have identical information volume.

The rest examples will further illustrate this conclusion.

Example 4.5:

Consider the frame of discernment be $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Let the mass function be $m_0(\{\theta_1\}) = m_0(\{\theta_2\}) = m_0(\{\theta_3\}) = \frac{1}{19}$, $m_0(\{\theta_1, \theta_2\}) = m_0(\{\theta_1, \theta_3\}) = m_0(\{\theta_2, \theta_3\}) = \frac{3}{19}$, $m_0(\{\theta_1, \theta_2, \theta_3\}) = \frac{7}{19}$, which is the BPA distribution of the maximum Deng entropy when the cardinal of the frame of discernment is 3.

The information volume of this mass function can be calculated by **Definition 3.1**. The convergence procedure of $H_i(m)$ is listed in Table 4.

According to Table 4, when $i = 16$, $H_i(m) - H_{i-1}(m) < 0.001$, which means that $H_i(m)$ finally converges to 6.469009. Hence, when the cardinal of the frame of discernment is 3, the information volume of the BPA distribution of the maximum Deng entropy is $H_{IV-mass}(m) = 6.469009$.

Example 4.6:

Consider the frame of discernment be $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Let the mass function be $m_0(\Theta) = m_0(\{\theta_1, \theta_2, \theta_3\}) = 1$, which is called the total uncertainty case when the cardinal of the frame of discernment is 3.

The information volume of the total uncertainty case can be calculated by **Definition 3.1**. The

Table 4: The convergence procedure of $H_i(m)$

i	$H_i(m)$	i	$H_i(m)$
1	4.247928	9	6.432107
2	5.127754	10	6.447290
3	5.661354	11	6.456402
4	5.983615	12	6.461869
5	6.177746	13	6.465150
6	6.294510	14	6.467119
7	6.364674	15	6.468300
8	6.406810	16	6.469009

convergence procedure of $H_i(m)$ is listed in Table 5.

Table 5: The convergence procedure of $H_i(m)$

i	$H_i(m)$	i	$H_i(m)$
1	2.807355	10	6.432107
2	4.247928	11	6.447290
3	5.127754	12	6.456402
4	5.661354	13	6.461869
5	5.983615	14	6.465150
6	6.177746	15	6.467119
7	6.294510	16	6.468300
8	6.364674	17	6.469009
9	6.406810		

According to Table 5, when $i = 17$, $H_i(m) - H_{i-1}(m) < 0.001$, which means that $H_i(m)$ finally converges to 6.469009. Hence the information volume of this total uncertainty case is $H_{IV-mass}(m) = 6.469009$.

Example 4.5 and **Example 4.6** further illustrate that, when the cardinal of the frame of discernment is identical, the information volume of the BPA distribution for the maximum Deng entropy is the same as that of the total uncertainty case. This point is consistent with the intuition.

5 Conclusion

In this paper, we define the information volume of a given mass function based on Deng entropy. In addition, some examples are shown for better understanding of proposed information volume fo mass function.

Some concluding remarks can be shown as follows.

- 1) If the mass function degenerates as probability distribution, the information volume is the same as Shannon entropy.
- 2) Given a mass function, the corresponding information volume is larger than its uncertainty measured by Deng entropy.
- 3) One interesting point is that, when the cardinal of the frame of discernment is identical, the BPA distribution for the maximum Deng entropy and the total uncertainty case has the same information volume. This point is coincide with the intuition.

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