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Observer-based feedback control of interval-valued fuzzy singular system with time-varying delay and stochastic faults

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Abstract

There are countless applications of non-linear systems that incorporate delay and algebraic equations. Despite current improvements in control theory, stochastic actuator defects still pose challenges when it comes to these systems. Furthermore, when it is not possible to measure the states of the system, and when uncertainties affect the system under investigation, the problem becomes even more complex. This paper is concerned with fault-tolerant observer-based controller synthesis for non-linear delayed singular systems with uncertainties and stochastic actuator failures. On the basis of interval valued models, a new Lyapunov-Krasovskii functional is built to develop a less conservative criterion to ensure that the closed-loop system is admissible in the mean-square sense. In addition, as these matrices are coupled with multiple variables, finding the parametric matrices of the observer and controller in terms of the obtained condition is more complex and challenging. The proposed method employs the matrix inequality decoupling technique to resolve this issue. Eventually, simulations are carried out to demonstrate the applicability of the proposed method.

Keywords: Singular system, (IVF) model , time varying delay, probabilistic faults, reliable observer-based control.

1 Introduction

This section includes the literature review, notations, and acronyms used in the document, as well as an outline of the publication and its goals.

1.1 Literature Review

Singular systems which are described by a couple of algebraic and differential equations, are characterized by their different modes namely finite dynamic modes, infinite non-dynamic modes, and infinite dynamic modes, respectively. The infinite dynamic modes has the feature to destroy the stability and the performances of the system. Thus, the admissibility, that includes stability, regularity, and non-impulsiveness/causality should be verified when dealing with this class of systems. As a consequence, the investigation of singular systems is both theoretically and practically important [1, 4, 39]. It is worth noting that time delays are common in many physical plants, and they can have a substantial negative impact on performance and even the stability of practical systems [7, 27, 32, 42, 43, 44]. Singular models and time-delay phenomena are general enough to enable some fundamental results from the theory of state-space systems to be extended to this class of systems (see for instance [3, 5, 10, 15, 17, 45]). On the other hand, the research on nonlinear systems [9] is an extremely hard issue due to their inherent complexity. Due to its rigorous mathematical structure, the T-S fuzzy model [14] has recently been applied to handle nonlinear complex systems, since this model has been known for its powerful approximation of smoothly nonlinear systems. In many cases, uncertainty arises such as partially unknown parameters, unpredictable disturbances in the system, varying interpretations of linguistic variables of the fuzzy models, etc. It is worth noting, however, that the membership functions of type-1 fuzzy sets are well-known, and the control problem can not be handled directly for nonlinear plants with parameter uncertainties. Research on (IVF) fuzzy systems has attracted attention due to the advantages of (IVF) sets over type 1 fuzzy sets in dealing with parameter uncertainties, and many control design results have been developed [24, 29]. Among others, in [33], the fuzzy decentralized output feedback control is investigated using the event-triggered approach for interval type-2 fuzzy systems against input saturation. The study in [18] discussed the issue of filter design for IVF systems with D-stability constraint. The authors in [13] investigated the discrete-time interval-valued fuzzy systems with actuator faults to study the reliable non-fragile control problem with H_∞ performance. Very recently, the IVF approach has been extended to non-linear singular systems with uncertainties. To mention a few, the admissibilization for IVF singular systems is studied in [6]. Based on the LMI approach, the dynamic output-feedback control design issue is investigated in [30] for singular interval-valued fuzzy systems.

We emphasize that, the evolution of industrial engineering has led to numerous practical plants becoming quite complex with failures. Control communities become concerned about this issue as actuator/sensor failures can adversely affect system performance. The goal of this issue is to introduce the concept of fault-tolerant control (FTC) and fault diagnosis as critical approaches for designing reliable controllers that are capable of maintaining the critical functionality of systems subject to problems and failures [47, 50]. Kavarian et al develop a method for designing fault-tolerant controllers for power systems subject to random changes and actuator failures in [12]. The FTC method for wind-diesel hybrid systems with time-varying bounded sensor faults has been proposed in [11]. In [37], the reliable observer-based control problem for discrete-time Takagi-Sugeno fuzzy systems with time-varying delay and stochastic actuator faults is formulated from the input-output approach. We also report on some results relating to FTC for singular systems. For systems with actuator and/or sensor faults, sliding mode control was used [25]. The reliable control problem for nonlinear singularly perturbed systems with random actuator failures is discussed in [38]. It is worth pointing out that, actuator faults sometimes manifest themselves in a stochastic manner since the faults may occur occasionally with a random phenomenon, and a deterministic model cannot adequately describe actuator faults. As such, it is more appropriate and significant to consider a fault factor that obeys a certain distribution of probabilities. Up till now, the FTC based on a stochastic scenario has been the subject of many published results. Among others, we cite, the authors in [21] proposed a resilient and reliable controller for Markovian jump systems subject to stochastic intermittent actuator faults and stochastic controller gain fluctuations. For a class of networked control systems with random occurred actuator failures, an FTC control scheme was designed in [36]. The authors in [31] utilized an observer to develop a reliable consensus protocol for a multiagent system against stochastic actuator failures. In [35], a reliable fuzzy H_∞ control law was synthesized and applied for permanent magnet synchronous motor suffered from random actuator faults We know that, to the best of our knowledge, the problem FTC for IVF systems employing a reliable controller with stochastic faults has not been completely addressed in the literature, which motivates us to carry out this study.

In addition, it is also emphasized that, there are numerous complex plants with non-linearities where the state variables are not accessible due to a variety of factors, such as the lack of sensors that are able to detect specific states, or the increasing number of sensors making the system more complicated. Various approaches have been used in output feedback design, including static output feedback [26], dynamic output feedback [16, 30], observer adaptive control [22, 23], and fuzzy observer-based approaches [20, 28, 41]. Designing observer-based controllers by the IVF T-S fuzzy models is a crucial issue that needs to be extensively explored so that many result have been recently published [8, 48, 49]. To the extent of our knowledge, a few research efforts have been made on IVF singular systems with unmeasurable premise variables, time-varying delay and randomly occurring actuator failures. This establishes the second motivation for the present work.

1.2 Objective and Outline

The discussion above has inspired us to study, in this study, the observer-based FTC problem for a class of uncertain non-linear singular systems against the random event of actuator failures. This article is noteworthy for the following features.

(i) As an alternative to existing control schemes developed for type-1 fuzzy singular systems with delay and actuator failure [38, 40, 46], this study addresses a novel reliable controller design for interval-valued fuzzy systems which may exhibit actuator faults represented by stochastic variables with Bernoulli distribution

(ii) Contrary to the work in [33], which considers a fuzzy decentralized observer-based event-triggered control for interval type-2 fuzzy systems assuming that the premise variables are measurable, this work supposes that the premise variables are unknown.

(iii) Unlike the existing studies where the Finsler lemma is used in [2] to design the controller and observer gains, this work formulate a feasible control strategy for the considered control problem using the decoupling matrix procedure.

After setting out the introduction and aims of the study, the paper is organized as follows: In Section 2, the model and assumptions as well as the problem characterization are provided. Our main findings are presented and discussed in Section 3. The focus of this section is on the development of a new delay-dependent stochastic admissibility criterion for the system under steady using a new Lyapunov-Krasovskii functional. In addition, the design of the controller and observer gains is carried out by employing the matrix inequality decoupling technique to resolve the bilinear matrix inequality problem. Section 4 presents numerical simulations as a mean of exhibiting the potential applications of the suggested control strategy and validating its significance. In Section 5, we present some conclusions regarding the obtained results and some suggestions for future research.

1.3 Notations

Table 1 lists the notations and acronyms that should be used in this study.

2 Preliminaries and Problem Statement

The aim of this section is to introduce some preliminaries that facilitate understanding of our proposal and state the problem we are investigating.

Table 1: List of notations and acronyms used in the paper.

Symbol	Acronym/Notation
$\mathbb{R}$	set of the real numbers
$\mathcal{X} \in \mathbb{R}^n$	n-dimensional Euclidean space
$\boldsymbol{X} \in \mathbb{R}^{n\times m}$	$n \times m$ real matrix
$\boldsymbol{X} > 0$	real symmetric positive definite matrix $\boldsymbol{X}$
$\Vert\boldsymbol{X}\Vert$	norm of the matrix $\boldsymbol{X}$
$\boldsymbol{X}^\top$	transpose of the matrix $\boldsymbol{X}$
$\mathrm{sym}(\boldsymbol{X})$	$\boldsymbol{X} + \boldsymbol{X}^\top$
$\lambda()$	eigenvalue of a matrix
$\mathbb{E}$	mathematical expectation
$*$	term that is induced by symmetry
r	number of `if-then` rules
LMI	linear matrix inequality
BMI	bi-linear matrix inequality
IVF	Interval-valued fuzzy
T-S	Takagi-Sugeno

2.1 IVF Model

Consider a class of non-linear singular system which can be described by the following IVF Model:

$$\mathbf{R}_i: \text{ If } \theta_1(\boldsymbol{x}(t)) \text{ is } \boldsymbol{\mathcal{M}}_i^1 \text{ and If } \theta_2(\boldsymbol{x}(t)) \text{ is } \boldsymbol{\mathcal{M}}_i^2 \cdots \text{If } \theta_s(\boldsymbol{x}(t)) \text{ is } \boldsymbol{\mathcal{M}}_i^s, \text{ Then}$$

$$\begin{cases} \boldsymbol{E}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{A}_{di}\boldsymbol{x}(t-d(t)) + \boldsymbol{B}_i\boldsymbol{u}^F(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}_i\boldsymbol{x}(t) \\ \boldsymbol{x}(t) = \boldsymbol{\phi}(t), \forall t \in [-d_2\ 0], \end{cases} \tag{1}$$

where $\boldsymbol{\mathcal{M}}_i^k$ is an IVF set of rule i corresponding to the premise variable $\theta_i(\boldsymbol{x}(t))$, $k = 1, 2, \cdots, s$; k is the number of premise variables, and $i \in \mathbb{S} \triangleq \{1, 2, \ldots, r\}$ is the number of rules. $\boldsymbol{x}(t) \in \boldsymbol{R}^n$, $\boldsymbol{y}(t) \in \boldsymbol{R}^{n_y}$, and $\boldsymbol{u}^F(t) \in \boldsymbol{R}^m$, define, respectively, the state, the output and input vectors. Matrices $\boldsymbol{A}_i$, $\boldsymbol{A}_{di}$, and $\boldsymbol{B}_i$ in model (1) are known with appropriate dimensions. $d(t)$ stands for the time varying delay, and $\boldsymbol{\phi}(t)$ defines the initial state for all $t \in [-d_2\ 0]$.

2.2 Assumptions and Resulting Model

A1 $d(t)$ is a continuous function such that

$$0 < d_1 \leq d(t) \leq d_2, \quad 0 \leq \dot{d}(t) \leq d_r \tag{2}$$

where d_1 represents the lower delay bound, d_2 stands for upper delay bounds, and d_r is the delay variation rate.

A2 Singular matrix $\boldsymbol{E}$ satisfies $rank(\boldsymbol{E}) = q < n$.

A3 The actuator fault has the following form:

$$\boldsymbol{u}^F(t) = \boldsymbol{\Gamma}\boldsymbol{u}(t) = \sum_{s=1}^{m} \gamma_s \boldsymbol{\Sigma}_s \boldsymbol{u}(t) \tag{3}$$

where $\boldsymbol{\Gamma} = \mathrm{diag}\{\gamma_1, \gamma_2, \cdots, \gamma_m\}$ is a diagonal matrix with m uncorrelated random variables, and $\boldsymbol{\Sigma}_s = \mathrm{diag}\{\underbrace{0, \cdots, 0}_{s-1}, 1, \underbrace{0, \cdots, 0}_{s-m}\}$.

Assume that $\gamma_s \in \{0,\ 1\}$ is a Bernoulli distributed stochastic variable with a probabilistic density function $p_s(\gamma_s)$, where the expectation and variance of γ_s are respectively, defined as $\bar{\gamma}_s$ and π_s. Define $\bar{\boldsymbol{\Gamma}} = \mathbb{E}\{\boldsymbol{\Gamma}\} = \mathrm{diag}\{\bar{\gamma}_1, \bar{\gamma}_2, \cdots, \bar{\gamma}_m\}$. It follows that for any matrix $\boldsymbol{Z} > 0$

$$\begin{cases} \mathbb{E}\Big\{(\boldsymbol{\Gamma} - \bar{\boldsymbol{\Gamma}})\Big\} & = \sum_{s=1}^{m} (\gamma_s - \bar{\gamma}_s)\boldsymbol{\Sigma}_s = 0 \\ \mathbb{E}\Big\{(\boldsymbol{\Gamma} - \bar{\boldsymbol{\Gamma}})^\top \boldsymbol{Z}(\boldsymbol{\Gamma} - \bar{\boldsymbol{\Gamma}})\Big\} & = \sum_{s=1}^{m} \pi_s^2 \boldsymbol{\Sigma}_s^\top \boldsymbol{Z}\boldsymbol{\Sigma}_s \end{cases} \tag{4}$$

Based on the IVF approach, the following interval defines the firing strength of the ith rule:

$$\mathbb{M}_i = \Big[\prod_{k=1}^{s} \underline{\omega}_{\boldsymbol{\mathcal{M}}_i^k(\boldsymbol{\theta}(\boldsymbol{x}(t)))} \quad \prod_{k=1}^{s} \bar{\omega}_{\boldsymbol{\mathcal{M}}_i^k(\boldsymbol{\theta}(\boldsymbol{x}(t)))}\Big] = \Big[\underline{\mu}_i(\boldsymbol{x}(t)) \quad \bar{\mu}_i(\boldsymbol{x}(t))\Big]$$

where $\underline{\mu}_i(\boldsymbol{x}(t)) \geq 0$, and $\bar{\mu}_i(\boldsymbol{x}(t)) \geq 0$ represent, respectively, the the lower and upper membership functions. Accordingly, $\underline{\omega}_{\boldsymbol{\mathcal{M}}_i^k(\boldsymbol{\theta}(\boldsymbol{x}(t)))} \geq 0$, and $\bar{\omega}_{\boldsymbol{\mathcal{M}}_i^k(\boldsymbol{\theta}(\boldsymbol{x}(t)))} \geq 0$ stands, respectively, for the lower and upper grades of membership. Therewith, the non-linear singular system can be described as

$$\begin{cases} \boldsymbol{E}\dot{\boldsymbol{x}}(t) = \displaystyle\sum_{i=1}^{r} \mu_i(\boldsymbol{x}(t))\Big(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{A}_{di}\boldsymbol{x}(t - d(t)) + \boldsymbol{B}_i\boldsymbol{u}^F(t)\Big) \\ \boldsymbol{y}(t) = \displaystyle\sum_{i=1}^{r} \mu_i(\boldsymbol{x}(t))\boldsymbol{C}_i\boldsymbol{x}(t), \end{cases} \tag{5}$$

$\mu_i(\boldsymbol{x}(t))$ denotes the grade of membership of the ith local system defined as

$$\mu_i(\boldsymbol{x}(t)) = \underline{\alpha}_i(\boldsymbol{x}(t))\underline{\mu}_i(\boldsymbol{x}(t)) + \bar{\alpha}_i(\boldsymbol{x}(t))\bar{\mu}_i(\boldsymbol{x}(t)),\ \sum_{i=1}^{r} \mu_i(\boldsymbol{x}(t)) = 1$$

where $\underline{\alpha}_i(\boldsymbol{x}(t))$ and $\bar{\alpha}_i(\boldsymbol{x}(t)$ are two weighting coefficient functions satisfying

$$0 \leq \underline{\alpha}_i(\boldsymbol{x}(t)), \bar{\alpha}_i(\boldsymbol{x}(t)) \leq 1,\ \underline{\alpha}_i(\boldsymbol{x}(t)) + \bar{\alpha}_i(\boldsymbol{x}(t)) = 1 \tag{6}$$

Note that, by introducing weighting coefficient functions, we can represent any time-variant or time-invariant unmeasured parameters of the general non-linear system. Moreover, these functions are not necessarily known but exist and satisfy (6).

2.3 Fuzzy Observer-based Controller

Generally, the system's states cannot be fully measured in practice. To estimate the state variables of system (5), an observer is required. We will proceed to address this issue by considering the following IVF observer for system (5):

$$\begin{aligned} &\mathbf{R}_i: \text{ If } \theta_1(\hat{\boldsymbol{x}}(t)) \text{ is } \boldsymbol{\mathcal{M}}_i^1 \text{ and If } \theta_2(\hat{\boldsymbol{x}}(t)) \text{ is } \boldsymbol{\mathcal{M}}_i^2 \cdots \text{If } \theta_s(\hat{\boldsymbol{x}}(t)) \text{ is } \boldsymbol{\mathcal{M}}_i^s, \text{ Then} \\ &\begin{cases} \boldsymbol{E}\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}_l\hat{\boldsymbol{x}}(t) + \boldsymbol{B}_l\boldsymbol{u}^F(t) + \boldsymbol{L}_l(\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)) \\ \hat{\boldsymbol{y}}(t) = \boldsymbol{C}_l\hat{\boldsymbol{x}}(t), \end{cases} \end{aligned}$$

where $\hat{\boldsymbol{x}}(t)$ denotes the state of the observer and $\hat{\boldsymbol{y}}(t)$ is the observer output. $\boldsymbol{L}_l$, $(l \in \mathbb{S})$ is the observer gain to be evaluated. The whole fuzzy observer can be inferred as follows:

$$\begin{cases} \boldsymbol{E}\dot{\hat{\boldsymbol{x}}}(t) = \displaystyle\sum_{l=1}^{r} \mu_l(\hat{\boldsymbol{x}}(t))\Big(\boldsymbol{A}_l\hat{\boldsymbol{x}}(t) + \boldsymbol{B}_l\ \boldsymbol{u}^F(t) + \boldsymbol{L}_l(\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t))\Big) \\ \hat{\boldsymbol{y}}(t) = \displaystyle\sum_{l=1}^{r} \mu_l(\hat{\boldsymbol{x}}(t))(\boldsymbol{C}_l\hat{\boldsymbol{x}}(t)) \end{cases} \tag{7}$$

To increase the flexibility of fuzzy controller design, we propose a controller that uses exclusive membership functions that would not be shared with (5). The state-feedback IVF controller has the following structure:

$$\begin{aligned} &\mathbf{R}_i: \text{ If } \vartheta_1(\hat{\boldsymbol{x}}(t)) \text{ is } \boldsymbol{\mathcal{N}}_i^1 \text{ and If } \vartheta_2(\boldsymbol{x}(t)) \text{ is } \boldsymbol{\mathcal{N}}_i^2 \cdots \text{If } \vartheta_s(\hat{\boldsymbol{x}}(t)) \text{ is } \boldsymbol{\mathcal{N}}_i^s, \text{ Then} \\ &\boldsymbol{u}(t) = \boldsymbol{K}_j\hat{\boldsymbol{x}}(t) \end{aligned}$$

where $\boldsymbol{\vartheta}(t) = [\vartheta_1(\hat{\boldsymbol{x}}(t)), \vartheta_2(\hat{\boldsymbol{x}}(t)), \ldots, \vartheta_s(\hat{\boldsymbol{x}}(t))]$ is the premise variable vector, $\boldsymbol{\mathcal{N}}_j^{k_c}$, $k_c = 1, 2, \cdots, s$ stands for the IVF sets, and $\boldsymbol{K}_j$ is the state feedback gain matrix of rule j.
Following is the firing interval for the jth rule:

$$\boldsymbol{\mathcal{N}}_i = \left[\underline{\nu}_j(\hat{\boldsymbol{x}}(t)) \quad \bar{\nu}_j(\hat{\boldsymbol{x}}(t))\right], \; i \in \mathbb{S}$$

where

$$\underline{\nu}_j(\hat{\boldsymbol{x}}(t)) = \prod_{k_c=1}^{s} \underline{\omega}_{\boldsymbol{\mathcal{N}}_i^{k}(\boldsymbol{\vartheta}(t))} \geq 0, \qquad \bar{\nu}_j(\hat{\boldsymbol{x}}(t)) = \prod_{k_c=1}^{s} \bar{\omega}_{\boldsymbol{\mathcal{N}}_i^{k_c}(\boldsymbol{\vartheta}(t))} \geq 0, \tag{8}$$

$\underline{\nu}_j(\hat{\boldsymbol{x}}(t))$ and $\bar{\nu}_j(\hat{\boldsymbol{x}}(t))$ represent,respectively, the lower and upper membership functions. $\underline{\omega}_{\boldsymbol{\mathcal{N}}_i^{k_c}(\boldsymbol{\vartheta}(t))} \geq 0$ and $\bar{\omega}_{\boldsymbol{\mathcal{N}}_i^{k_c}(\boldsymbol{\vartheta}(t))} \geq 0$ are the lower and upper grades of membership of $\boldsymbol{\theta}(t)$ in $\boldsymbol{\mathcal{N}}_i^{j}$, respectively. The global fuzzy model will be defined as follows:

$$\boldsymbol{u}(t) = \sum_{j=1}^{r} \nu_j(\hat{\boldsymbol{x}}(t))(\boldsymbol{K}_j\hat{\boldsymbol{x}}(t)) \tag{9}$$

$$\nu_j(\hat{\boldsymbol{x}}(t)) = \frac{\underline{\beta}_j(\hat{\boldsymbol{x}}(t))\underline{\nu}_j(\hat{\boldsymbol{x}}(t)) + \bar{\beta}_j(\hat{\boldsymbol{x}}(t))\bar{\nu}_j(\hat{\boldsymbol{x}}(t))}{\sum_{l=1}^{r}(\underline{\beta}_l(\hat{\boldsymbol{x}}(t))\underline{\nu}_l(\hat{\boldsymbol{x}}(t)) + \bar{\beta}_l(\hat{\boldsymbol{x}}(t))\bar{\nu}_l(\hat{\boldsymbol{x}}(t)))}, \; \nu_j(\hat{\boldsymbol{x}}(t)) \geq 0, \; \sum_{i=1}^{r}\nu_j(\hat{\boldsymbol{x}}(t)) = 1 \tag{10}$$

With the error $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$, together with (5), (7), and (9) we obtain the following closed-loop system:

$$\tilde{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\mu_i\hat{\nu}_j\hat{\mu}_l\Big(\tilde{\boldsymbol{A}}_{ijl}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_{di}\tilde{\boldsymbol{x}}(t - d(t)) + \tilde{\boldsymbol{B}}_{il}\tilde{\boldsymbol{\Gamma}}\tilde{\boldsymbol{K}}_j\tilde{\boldsymbol{x}}(t)\Big) \tag{11}$$

where $\tilde{\boldsymbol{x}}(t) = [\boldsymbol{x}^\top(t), \; \boldsymbol{e}^\top(t)]^\top$, $\tilde{\boldsymbol{\Gamma}} = \boldsymbol{\Gamma} - \bar{\boldsymbol{\Gamma}}$, and

$$\tilde{\boldsymbol{A}}_{ijl} = \begin{bmatrix} \boldsymbol{A}_i + \boldsymbol{B}_i\bar{\boldsymbol{\Gamma}}\boldsymbol{K}_j & -\boldsymbol{B}_i\bar{\boldsymbol{\Gamma}}\boldsymbol{K}_j \\ (\boldsymbol{A}_i - \boldsymbol{A}_l) + (\boldsymbol{B}_i - \boldsymbol{B}_l)\bar{\boldsymbol{\Gamma}}\boldsymbol{K}_j - \boldsymbol{L}_l(\boldsymbol{C}_i - \boldsymbol{C}_l) & \boldsymbol{A}_i - (\boldsymbol{B}_i - \boldsymbol{B}_l)\bar{\boldsymbol{\Gamma}}\boldsymbol{K}_j - \boldsymbol{L}_l\boldsymbol{C}_l \end{bmatrix}, \quad \tilde{\boldsymbol{A}}_{di} = \begin{bmatrix} \boldsymbol{A}_{di} & 0 \\ \boldsymbol{A}_{di} & 0 \end{bmatrix}$$

$$\tilde{\boldsymbol{B}}_{il} = \begin{bmatrix} \boldsymbol{B}_i \\ (\boldsymbol{B}_i - \boldsymbol{B}_l) \end{bmatrix}, \; \tilde{\boldsymbol{K}}_j = \begin{bmatrix} \boldsymbol{K}_j & -\boldsymbol{K}_j \end{bmatrix} \qquad \tilde{\boldsymbol{E}} = \begin{bmatrix} \boldsymbol{E} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{E} \end{bmatrix}$$

Remark 1. *Generally, the fuzzy observers and fuzzy controllers based on type-1 fuzzy share the same membership functions of the model. However, the membership functions used to describe the IVF systems are unknown except for their bounds. Thus, in this study, the membership functions of fuzzy observers are calculated based on the estimated state variable $\hat{x}(t)$ rather than $x(t)$.*

2.4 Problem Statement

It is our primary objective in this paper to design an IVF observer-based controller that maintains the closed-loop system admissible in the event of random actuator failure in non-linear singular systems expressed by the IVF model as specified in (5). Let us begin by stating the following lemma that will be used in our main results.

Lemma 2. *[34] Let $\boldsymbol{P}$ be a symmetric matrix such that $\boldsymbol{E}_L^\top\boldsymbol{P}\boldsymbol{E}_L > 0$. Then, matrix $\boldsymbol{P}\boldsymbol{E} + \boldsymbol{U}^\top\boldsymbol{X}V^\top$ is non singular so that*

$$(\boldsymbol{P}\boldsymbol{E} + \boldsymbol{U}^\top\boldsymbol{X}\boldsymbol{V}^\top)^{-1} = \bar{\boldsymbol{P}}\boldsymbol{E}^\top + \boldsymbol{V}\bar{\boldsymbol{X}}\boldsymbol{U}$$

where $\bar{\boldsymbol{P}} \in \mathbb{R}^{n\times n}$ is a symmetric matrix verifying $\boldsymbol{E}_R^\top\bar{\boldsymbol{P}}\boldsymbol{E}_R = (\boldsymbol{E}_L^\top\boldsymbol{P}\boldsymbol{E}_L)^{-1}$, $\bar{\boldsymbol{X}} \in \mathbb{R}^{(n-q)\times(n-q)}$ is a non singular matrix satisfying $\bar{\boldsymbol{X}} = (\boldsymbol{V}^\top\boldsymbol{V})^{-1}\boldsymbol{V}^{-1}(\boldsymbol{U}\boldsymbol{U}^\top)^{-1}$, $\boldsymbol{U}$ be a full row rank matrix so that $\boldsymbol{U}\boldsymbol{E} = 0$, and $\boldsymbol{V}$ be a full column rank matrix such that $\boldsymbol{E}V = 0$. $\boldsymbol{E}_L$ and $\boldsymbol{E}_R$ are full column rank matrices with $\boldsymbol{E} = \boldsymbol{E}_L\boldsymbol{E}_R^\top$.

3 Main Results

3.1 Admissibility Analysis

As we will see below, we are able to obtain sufficient delay-dependent conditions for the closed-loop system stated in (11) to be stochastically admissible.

Theorem 3. *For given constants d_1, d_2 and $0 \le d_r < 1$, assume (A1) holds. System (11) is admissible, if there exist positive matrices $\boldsymbol{P}_k \in \mathbb{R}^{n\times n}$, $\tilde{\boldsymbol{Q}} \in \mathbb{R}^{2n\times 2n}$, $\tilde{\boldsymbol{R}}_1 \in \mathbb{R}^{2n\times 2n}$, $\tilde{\boldsymbol{R}}_2 \in \mathbb{R}^{2n\times 2n}$, $\tilde{\boldsymbol{R}}_3 > 0$, $\tilde{\boldsymbol{Q}}_a = \begin{bmatrix} \tilde{\boldsymbol{Q}}_{11} & \tilde{\boldsymbol{Q}}_{12} \\ \tilde{\boldsymbol{Q}}_{12}^\top & \tilde{\boldsymbol{Q}}_{22} \end{bmatrix} \in \mathbb{R}^{4n\times 4n}$, $\tilde{\boldsymbol{S}}_a = \begin{bmatrix} \tilde{\boldsymbol{S}}_{11} & \tilde{\boldsymbol{S}}_{12} \\ \tilde{\boldsymbol{S}}_{12}^\top & \tilde{\boldsymbol{S}}_{22} \end{bmatrix} \in \mathbb{R}^{4n\times 4n}$, non-singular matrices $\boldsymbol{X}_k \in \mathbb{R}^{n\times n}$, $\tilde{\boldsymbol{M}}_k$, $\tilde{\boldsymbol{N}}_k$, $\tilde{\boldsymbol{T}}_k$, and $\tilde{\boldsymbol{V}}_k$, $(k = 1, 2)$ such that the following inequality holds :*

$$\tilde{\boldsymbol{\Psi}}_{ijl}(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{A}}_{ijl}, \tilde{\boldsymbol{A}}_{di}) = \begin{bmatrix} \tilde{\boldsymbol{\psi}}_{11ijl}(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{A}}_{ijl}, \tilde{\boldsymbol{A}}_{di}) & \sqrt{\frac{d_1}{2}}\tilde{\boldsymbol{\psi}}_{ijl}\tilde{\boldsymbol{R}}_1 & \sqrt{\frac{d_{12}}{2}}\tilde{\boldsymbol{\psi}}_{ijl}\tilde{\boldsymbol{R}}_2 & \sqrt{d_{12}}\tilde{\boldsymbol{\psi}}_{ijl}\tilde{\boldsymbol{R}}_3 & \tilde{\boldsymbol{\psi}}_{15i} & \boldsymbol{I}_\pi \otimes \tilde{\boldsymbol{\psi}}_{16ijl} \\ * & -\tilde{\boldsymbol{R}}_1 & 0 & 0 & 0 & 0 \\ * & * & -\tilde{\boldsymbol{R}}_2 & 0 & 0 & 0 \\ * & * & * & -\tilde{\boldsymbol{R}}_3 & 0 & 0 \\ * & * & * & * & -\tilde{\boldsymbol{\psi}}_{55} & 0 \\ * & * & * & * & * & -\tilde{\boldsymbol{\psi}}_{66} \end{bmatrix} < 0 \tag{12}$$

where $d_{12} = d_2 - d_1$, $\tilde{\boldsymbol{R}} = \dfrac{d_1}{2}\tilde{\boldsymbol{R}}_1 + \dfrac{d_{12}}{2}\tilde{\boldsymbol{R}}_2 + d_{12}\tilde{\boldsymbol{R}}_3$, $\boldsymbol{\Pi}_1 = \boldsymbol{P}_1\boldsymbol{E} + \boldsymbol{U}^\top\boldsymbol{X}_1\boldsymbol{V}^\top$, $\boldsymbol{\Pi}_2 = \boldsymbol{P}_2\boldsymbol{E} + \boldsymbol{U}^\top\boldsymbol{X}_2\boldsymbol{V}^\top$,

$$\tilde{\boldsymbol{\psi}}_{11ijl}(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{A}}_{ijl}, \tilde{\boldsymbol{A}}_{di}) = \begin{bmatrix} \tilde{\boldsymbol{\Psi}}_{11i} & \tilde{\boldsymbol{\Psi}}_{12i} & \tilde{\boldsymbol{\Psi}}_{13i} & 0 & \tilde{\boldsymbol{S}}_{12} & 0 \\ * & \tilde{\boldsymbol{\Psi}}_{22i} & 0 & \tilde{\boldsymbol{\Psi}}_{24i} & 0 & \tilde{\boldsymbol{\Psi}}_{26i} \\ * & * & \tilde{\boldsymbol{\Psi}}_{33i} & -\tilde{\boldsymbol{Q}}_{12} & \tilde{\boldsymbol{\Psi}}_{35i} & 0 \\ * & * & * & \tilde{\boldsymbol{\Psi}}_{44i} & 0 & 0 \\ * & * & * & * & \tilde{\boldsymbol{\Psi}}_{55i} & -\tilde{\boldsymbol{S}}_{12} \\ * & * & * & * & * & \tilde{\boldsymbol{\Psi}}_{66i} \end{bmatrix}$$

$$\begin{aligned}
\tilde{\boldsymbol{\psi}}_{ijl} &= \mathrm{col}\left\{\tilde{\boldsymbol{A}}_{ijl}^\top \quad \tilde{\boldsymbol{A}}_{di}^\top \quad 0 \quad 0 \quad 0 \quad 0\right\} \\
\tilde{\boldsymbol{\psi}}_{15} &= \left[\tilde{\boldsymbol{\psi}}_{151} \quad \tilde{\boldsymbol{\psi}}_{152} \quad \tilde{\boldsymbol{\psi}}_{153} \quad \tilde{\boldsymbol{\psi}}_{154}\right] \\
\tilde{\boldsymbol{\psi}}_{16ijl} &= \left[\pi_1(\tilde{\boldsymbol{B}}_{il}\boldsymbol{\Sigma}_1\tilde{\boldsymbol{K}}_j)^\top, \ldots, \pi_m(\tilde{\boldsymbol{B}}_{il}\boldsymbol{\Sigma}_m\tilde{\boldsymbol{K}}_j)^\top\right] \\
\tilde{\boldsymbol{\psi}}_{55} &= \mathrm{diag}\left\{\tilde{\boldsymbol{R}}_1, \ \tilde{\boldsymbol{R}}_2, \ \tilde{\boldsymbol{R}}_3, \ \tilde{\boldsymbol{R}}_3\right\} \\
\tilde{\boldsymbol{\psi}}_{66} &= \mathrm{diag}\left\{\tilde{\boldsymbol{R}}^{-1}, \ \cdots, \ \tilde{\boldsymbol{R}}^{-1}\right\}, \\
\tilde{\boldsymbol{\psi}}_{151} &= \sqrt{\frac{d_1}{2}}\,\mathrm{col}\left\{\tilde{\boldsymbol{E}}\tilde{\boldsymbol{M}}_1 \quad 0 \quad \tilde{\boldsymbol{E}}\tilde{\boldsymbol{M}}_2 \quad 0 \quad 0 \quad 0\right\} \\
\tilde{\boldsymbol{\psi}}_{152} &= \sqrt{\frac{d_{12}}{2}}\,\mathrm{col}\left\{0 \quad 0 \quad \tilde{\boldsymbol{E}}\tilde{\boldsymbol{N}}_1 \quad 0 \quad \tilde{\boldsymbol{E}}\tilde{\boldsymbol{N}}_2 \quad 0\right\}, \\
\tilde{\boldsymbol{\psi}}_{153} &= \sqrt{d_{12}}\,\mathrm{col}\left\{0 \quad \tilde{\boldsymbol{E}}\tilde{\boldsymbol{T}}_1 \quad 0 \quad 0 \quad 0 \quad \tilde{\boldsymbol{E}}\tilde{\boldsymbol{T}}_2\right\} \\
\tilde{\boldsymbol{\psi}}_{154} &= \sqrt{d_{12}}\,\mathrm{col}\left\{0 \quad \tilde{\boldsymbol{E}}\tilde{\boldsymbol{V}}_2 \quad 0 \quad \tilde{\boldsymbol{E}}\tilde{\boldsymbol{V}}_1 \quad 0 \quad 0\right\} \\
\boldsymbol{I}_\pi &= \mathrm{col}\left\{I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right\}
\end{aligned}$$

$$\begin{aligned}
\tilde{\boldsymbol{\Psi}}_{11i} &= \tilde{\boldsymbol{Q}}_{11} + \tilde{\boldsymbol{S}}_{11} + \tilde{\boldsymbol{Q}}_1 + \mathrm{sym}(\tilde{\boldsymbol{M}}_1\tilde{\boldsymbol{E}}) + \mathrm{sym}(\tilde{\boldsymbol{\Pi}}^\top\tilde{\boldsymbol{A}}_{ijl}) \\
\tilde{\boldsymbol{\Psi}}_{12i} &= (\tilde{\boldsymbol{\Pi}}^\top\tilde{\boldsymbol{A}}_{di}) \\
\tilde{\boldsymbol{\Psi}}_{13i} &= \tilde{\boldsymbol{Q}}_{12} - \tilde{\boldsymbol{M}}_1\tilde{\boldsymbol{E}} + (\tilde{\boldsymbol{M}}_2\tilde{\boldsymbol{E}})^\top \\
\tilde{\boldsymbol{\Psi}}_{15i} &= -\tilde{\boldsymbol{N}}_1\tilde{\boldsymbol{E}} + (\tilde{\boldsymbol{N}}_2\tilde{\boldsymbol{E}})^\top + \tilde{\boldsymbol{S}}_{12} \\
\tilde{\boldsymbol{\Psi}}_{22i} &= -(1 - h_d)\tilde{\boldsymbol{Q}}_1 + \mathrm{sym}(\tilde{\boldsymbol{T}}_1\tilde{\boldsymbol{E}}) - \mathrm{sym}(\tilde{\boldsymbol{V}}_2\tilde{\boldsymbol{E}}) \\
\tilde{\boldsymbol{\Psi}}_{24i} &= -\tilde{\boldsymbol{V}}_2\tilde{\boldsymbol{E}} + (\tilde{\boldsymbol{V}}_1\tilde{\boldsymbol{E}})^\top \\
\tilde{\boldsymbol{\Psi}}_{26i} &= -\tilde{\boldsymbol{T}}_1\tilde{\boldsymbol{E}} + (\tilde{\boldsymbol{T}}_2\tilde{\boldsymbol{E}})^\top \\
\tilde{\boldsymbol{\Psi}}_{33i} &= \tilde{\boldsymbol{Q}}_{22} - \tilde{\boldsymbol{Q}}_{11} - \mathrm{sym}(\tilde{\boldsymbol{M}}_2\tilde{\boldsymbol{E}}) + \mathrm{sym}(\tilde{\boldsymbol{N}}_1\tilde{\boldsymbol{E}}) \\
\tilde{\boldsymbol{\Psi}}_{35i} &= -\tilde{\boldsymbol{N}}_1\tilde{\boldsymbol{E}} + \tilde{\boldsymbol{N}}_2\tilde{\boldsymbol{E}} \\
\tilde{\boldsymbol{\Psi}}_{44i} &= -\tilde{\boldsymbol{Q}}_{22} + \mathrm{sym}(\tilde{\boldsymbol{V}}_1\tilde{\boldsymbol{E}}) \\
\tilde{\boldsymbol{\Psi}}_{55i} &= \tilde{\boldsymbol{S}}_{22} - \tilde{\boldsymbol{S}}_{11} - \mathrm{sym}(\tilde{\boldsymbol{N}}_2\tilde{\boldsymbol{E}}) \\
\tilde{\boldsymbol{\Psi}}_{66i} &= -\tilde{\boldsymbol{S}}_{22} - \mathrm{sym}(\tilde{\boldsymbol{T}}_2\tilde{\boldsymbol{E}}) \\
\tilde{\boldsymbol{\Pi}} &= \mathrm{diag}\left\{\boldsymbol{\Pi}_1, \ \boldsymbol{\Pi}_2\right\}
\end{aligned}$$

Matrices $\boldsymbol{U} \in \mathbb{R}^{(n-q)\times(n)}$ and $\boldsymbol{V} \in \mathbb{R}^{(n)\times(n-q)}$ are of full rank such that $\boldsymbol{U}\boldsymbol{E} = 0$ and $\boldsymbol{E}\boldsymbol{V} = 0$.

Proof. First, we will prove the stability of system (11). For this purpose, we choose the following

Lyapunov-Krasovsky functional:

$$\begin{aligned} V(\tilde{\boldsymbol{x}}(t)) &= V_1(\tilde{\boldsymbol{x}}(t)) + V_2(\tilde{\boldsymbol{x}}(t)) + V_3(\tilde{\boldsymbol{x}}(t)) \\ V_1(\tilde{\boldsymbol{x}}(t)) &= \tilde{\boldsymbol{x}}^T(t)\tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{P}}\tilde{\boldsymbol{E}}\tilde{\boldsymbol{x}}(t) \\ V_2(\tilde{\boldsymbol{x}}(t)) &= \int_{t-\frac{d_1}{2}}^{t} \tilde{\boldsymbol{\zeta}}_1^\top(s)\tilde{\boldsymbol{Q}}_a\tilde{\boldsymbol{\zeta}}_1(s)ds + \int_{t-\frac{d_2}{2}}^{t} \tilde{\boldsymbol{\zeta}}_2^\top(s)\tilde{\boldsymbol{S}}_a\tilde{\boldsymbol{\zeta}}_2^\top(s)ds + \int_{t-d(t)}^{t} \tilde{\boldsymbol{x}}^\top(s)\boldsymbol{Q}\tilde{\boldsymbol{x}}(s)ds \\ V_3(\tilde{\boldsymbol{x}}(t)) &= \int_{-\frac{d_1}{2}}^{0}\int_{t+\theta}^{t} \dot{\tilde{\boldsymbol{x}}}^\top(s)\tilde{\boldsymbol{E}}^\top\tilde{\boldsymbol{R}}_1\tilde{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)dsd\theta + \int_{-\frac{d_2}{2}}^{-\frac{d_1}{2}}\int_{t+\theta}^{t} \dot{\tilde{\boldsymbol{x}}}^\top(s)\tilde{\boldsymbol{E}}^\top\tilde{\boldsymbol{R}}_2\tilde{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)dsd\theta \\ &\quad + \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{\tilde{\boldsymbol{x}}}^\top(s)\tilde{\boldsymbol{E}}^\top\tilde{\boldsymbol{R}}_3\tilde{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)dsd\theta \end{aligned} \tag{13}$$

where

$$\tilde{\boldsymbol{\zeta}}_1(t) = \begin{bmatrix}\tilde{\boldsymbol{x}}^T(t) & \tilde{\boldsymbol{x}}^\top(t-\frac{d_1}{2})\end{bmatrix}^\top, \quad \tilde{\boldsymbol{\zeta}}_2(t) = \begin{bmatrix}\tilde{\boldsymbol{x}}^T(t) & \tilde{\boldsymbol{x}}^\top(t-\frac{d_2}{2})\end{bmatrix}^\top,$$

Define the infinitesimal operator $\mathcal{L}$ of $V(\tilde{\boldsymbol{x}}(t))$ as follows:

$$\mathcal{L}(V(\tilde{\boldsymbol{x}}(t))) = \lim_{\Delta\longrightarrow 0^+}\frac{1}{\Delta}\{\mathbb{E}\{V(\tilde{\boldsymbol{x}}(t+\Delta))|\tilde{\boldsymbol{x}}(t)\} - V(\tilde{\boldsymbol{x}}(t))\} \tag{14}$$

Evaluating the derivative of $V(\tilde{\boldsymbol{x}}(t))$ along the solutions of system (11), and noting $\boldsymbol{UE} = 0$, it results in

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V_1(\tilde{\boldsymbol{x}}(t))\} &= \mathbb{E}\{2\tilde{\boldsymbol{x}}^T(t)\tilde{\boldsymbol{E}}^\top\tilde{\boldsymbol{P}}\tilde{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(t)\} \\ &= 2\tilde{\boldsymbol{x}}^T(t)\tilde{\boldsymbol{\Pi}}^\top(\tilde{\boldsymbol{A}}_{ijl}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_{di}\tilde{\boldsymbol{x}}(t-d(t)) + \tilde{\boldsymbol{B}}_{il}\tilde{\boldsymbol{\Gamma}}\tilde{\boldsymbol{K}}_j\tilde{\boldsymbol{x}}(t)) \\ &= 2\tilde{\boldsymbol{x}}^T(t)\tilde{\boldsymbol{\Pi}}^\top(\tilde{\boldsymbol{A}}_{ijl}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_{di}\tilde{\boldsymbol{x}}(t-d(t))) \\ \mathbb{E}\{\mathcal{L}V_2(\tilde{\boldsymbol{x}}(t))\} &\le \tilde{\boldsymbol{\zeta}}_1^\top(t)\tilde{\boldsymbol{Q}}_a\tilde{\boldsymbol{\zeta}}_1(t) - \tilde{\boldsymbol{\zeta}}_1^\top(t-\frac{d_1}{2})\tilde{\boldsymbol{Q}}_a\tilde{\boldsymbol{\zeta}}_1(t-\frac{d_1}{2}) + \tilde{\boldsymbol{\zeta}}_2^\top(t)\tilde{\boldsymbol{S}}_a\tilde{\boldsymbol{\zeta}}_2(t) - \tilde{\boldsymbol{\zeta}}_2^\top(t-\frac{d_2}{2})\tilde{\boldsymbol{S}}_a\tilde{\boldsymbol{\zeta}}_2(t-\frac{d_2}{2}) \\ &\quad + \tilde{\boldsymbol{x}}^T(t)\tilde{\boldsymbol{Q}}_1\tilde{\boldsymbol{x}}(t) - (1-h_d)\tilde{\boldsymbol{x}}^\top(t-d(t))\tilde{\boldsymbol{Q}}_1\tilde{\boldsymbol{x}}(t-d(t)) \\ \mathbb{E}\{\mathcal{L}V_3(\tilde{\boldsymbol{x}}(t))\} &= \frac{d_1}{2}(\tilde{\boldsymbol{A}}_{ijl}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_{di}\tilde{\boldsymbol{x}}(t-d(t)))^\top\tilde{\boldsymbol{R}}_1(\tilde{\boldsymbol{A}}_{ijl}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_{di}\tilde{\boldsymbol{x}}(t-d(t))) \\ &\quad + \frac{d_{12}}{2}(\tilde{\boldsymbol{A}}_{ijl}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_{di}\tilde{\boldsymbol{x}}(t-d(t)))^\top\tilde{\boldsymbol{R}}_2(\tilde{\boldsymbol{A}}_{ijl}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_{di}\tilde{\boldsymbol{x}}(t-d(t))) \\ &\quad + d_{12}(\tilde{\boldsymbol{A}}_{ijl}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_{di}\tilde{\boldsymbol{x}}(t-d(t)))^\top\tilde{\boldsymbol{R}}_3(\tilde{\boldsymbol{A}}_{ijl}\tilde{\boldsymbol{x}}(t) + \tilde{\boldsymbol{A}}_{di}\tilde{\boldsymbol{x}}(t-d(t))) \\ &\quad + \mathbb{E}\Big\{(\tilde{\boldsymbol{B}}_{il}\tilde{\boldsymbol{\Gamma}}\tilde{\boldsymbol{K}}_j\tilde{\boldsymbol{x}}(t))^\top\tilde{\boldsymbol{R}}(\tilde{\boldsymbol{B}}_{il}\tilde{\boldsymbol{\Gamma}}\tilde{\boldsymbol{K}}_j\tilde{\boldsymbol{x}}(t))\Big\} \\ &\quad - \int_{t-\frac{d_1}{2}}^{t}\dot{\tilde{\boldsymbol{x}}}^\top(s)\tilde{\boldsymbol{E}}^\top\tilde{\boldsymbol{R}}_1\tilde{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)ds - \int_{t-\frac{d_2}{2}}^{t-\frac{d_1}{2}}\dot{\tilde{\boldsymbol{x}}}^\top(s)\tilde{\boldsymbol{E}}^\top\tilde{\boldsymbol{R}}_2\tilde{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)ds - \int_{t-d_2}^{t-d_1}\dot{\tilde{\boldsymbol{x}}}^\top(s)\tilde{\boldsymbol{E}}^\top\tilde{\boldsymbol{R}}_3\tilde{\boldsymbol{E}}\dot{\tilde{\boldsymbol{x}}}(s)ds \end{aligned} \tag{15}$$

Let $\tilde{\boldsymbol{\zeta}}_3(t) = \begin{bmatrix}\tilde{\boldsymbol{x}}^\top(t-\frac{d_1}{2}) & \tilde{\boldsymbol{x}}^\top(t-\frac{d_2}{2})\end{bmatrix}^\top$, $\tilde{\boldsymbol{\zeta}}_4(t) = \begin{bmatrix}\tilde{\boldsymbol{x}}^\top(t-d(t)) & \tilde{\boldsymbol{x}}^\top(t-d_2)\end{bmatrix}^\top$ $\tilde{\boldsymbol{\zeta}}_5(t) = \begin{bmatrix}\tilde{\boldsymbol{x}}^\top(t-d_1) & \tilde{\boldsymbol{x}}^\top(t-d(t))\end{bmatrix}^\top$, $\tilde{\boldsymbol{M}} = \begin{bmatrix}\tilde{\boldsymbol{M}}_1^\top & \tilde{\boldsymbol{M}}_2^\top\end{bmatrix}^\top$, $\tilde{\boldsymbol{N}} = \begin{bmatrix}\tilde{\boldsymbol{N}}_1^\top & \tilde{\boldsymbol{N}}_2^\top\end{bmatrix}^\top$, $\tilde{\boldsymbol{T}} = \begin{bmatrix}\tilde{\boldsymbol{T}}_1^\top & \tilde{\boldsymbol{T}}_2^\top\end{bmatrix}^\top$, and $\tilde{\boldsymbol{V}} = \begin{bmatrix}\tilde{\boldsymbol{V}}_1^\top & \tilde{\boldsymbol{V}}_2^\top\end{bmatrix}^\top$. By defining the following expression:

$$\int_{t-\frac{d_1}{2}}^{t}\begin{bmatrix}\tilde{\boldsymbol{E}}\dot{\boldsymbol{x}}(s)\\ \boldsymbol{\xi}_1(t)\end{bmatrix}^\top\begin{bmatrix}\tilde{\boldsymbol{R}}_1 & \tilde{\boldsymbol{M}}\\ * & \tilde{\boldsymbol{M}}\tilde{\boldsymbol{E}}\tilde{\boldsymbol{R}}_1^{-1}\tilde{\boldsymbol{E}}^\top\tilde{\boldsymbol{M}}^\top\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{E}}\dot{\boldsymbol{x}}(s)\\ \boldsymbol{\xi}_1(t)\end{bmatrix}ds \ge 0, \tag{16}$$

$$\int_{t-\frac{d_2}{2}}^{t-\frac{d_1}{2}}\begin{bmatrix}\tilde{\boldsymbol{E}}\dot{\boldsymbol{x}}(s)\\ \boldsymbol{\xi}_2(t)\end{bmatrix}^\top\begin{bmatrix}\tilde{\boldsymbol{R}}_2 & \tilde{\boldsymbol{N}}\\ * & \tilde{\boldsymbol{N}}\tilde{\boldsymbol{E}}\tilde{\boldsymbol{R}}_2^{-1}\tilde{\boldsymbol{E}}^\top\tilde{\boldsymbol{N}}^\top\end{bmatrix}\begin{bmatrix}\tilde{\boldsymbol{E}}\dot{\boldsymbol{x}}(s)\\ \boldsymbol{\xi}_2(t)\end{bmatrix}ds \ge 0 \tag{17}$$

we know that

$$-\int_{t-\frac{d_1}{2}}^{t} \dot{\boldsymbol{x}}(s)^\top \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{R}}_1 \tilde{\boldsymbol{E}} \dot{\boldsymbol{x}}(s) ds \le 2\boldsymbol{\xi}_1^\top(t) \tilde{\boldsymbol{M}} \Big[\tilde{\boldsymbol{E}} \quad -\tilde{\boldsymbol{E}}\Big] \boldsymbol{\xi}_1(t) + \frac{d_1}{2} \boldsymbol{\xi}_1^\top(t) \tilde{\boldsymbol{M}} \tilde{\boldsymbol{E}} \tilde{\boldsymbol{R}}_1^{-1} \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{M}}^\top \boldsymbol{\xi}_1(t) \tag{18}$$

$$-\int_{t-\frac{d_2}{2}}^{t-\frac{d_1}{2}} \dot{\boldsymbol{x}}(s)^\top \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{R}}_2 \tilde{\boldsymbol{E}} \dot{\boldsymbol{x}}(s) ds \le 2\boldsymbol{\xi}_3^\top(t) \tilde{\boldsymbol{N}} \Big[\tilde{\boldsymbol{E}} \quad -\tilde{\boldsymbol{E}}\Big] \boldsymbol{\xi}_3(t) + \frac{d_1}{2} \boldsymbol{\xi}_3^\top(t) \tilde{\boldsymbol{N}} \tilde{\boldsymbol{E}} \tilde{\boldsymbol{R}}_2^{-1} \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{N}}^\top \boldsymbol{\xi}_3(t) \tag{19}$$

Moreover, we have

$$-\int_{t-d_2}^{t-d_1} \dot{\tilde{\boldsymbol{x}}}^\top(s) \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{R}}_3 \tilde{\boldsymbol{E}} \dot{\tilde{\boldsymbol{x}}}(s) ds = -\int_{t-d_2}^{t-d(t)} \dot{\tilde{\boldsymbol{x}}}^\top(s) \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{R}}_3 \tilde{\boldsymbol{E}} \dot{\tilde{\boldsymbol{x}}}(s) ds - \int_{t-d(t)}^{t-d_1} \dot{\tilde{\boldsymbol{x}}}^\top(s) \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{R}}_3 \tilde{\boldsymbol{E}} \dot{\tilde{\boldsymbol{x}}}(s) ds \tag{20}$$

and

$$\begin{aligned} -\int_{t-d_2}^{t-d(t)} \dot{\boldsymbol{x}}(s)^\top \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{R}}_3 \tilde{\boldsymbol{E}} \dot{\boldsymbol{x}}(s) ds &\le 2\boldsymbol{\xi}_4^\top(t) \tilde{\boldsymbol{T}} \Big[\tilde{\boldsymbol{E}} \quad -\tilde{\boldsymbol{E}}\Big] \boldsymbol{\xi}_4(t) + (d_2 - d_1) \boldsymbol{\xi}_4^\top(t) \tilde{\boldsymbol{T}} \tilde{\boldsymbol{E}} \tilde{\boldsymbol{R}}_3^{-1} \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{T}}^\top \boldsymbol{\xi}_4(t) \\ -\int_{t-d(t)}^{t-d_1} \dot{\boldsymbol{x}}(s)^\top \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{R}}_3 \tilde{\boldsymbol{E}} \dot{\boldsymbol{x}}(s) ds &\le 2\boldsymbol{\xi}_5^\top(t) \tilde{\boldsymbol{V}} \Big[\tilde{\boldsymbol{E}} \quad -\tilde{\boldsymbol{E}}\Big] \boldsymbol{\xi}_5(t) + (d_2 - d_1) \boldsymbol{\xi}_5^\top(t) \tilde{\boldsymbol{V}} \tilde{\boldsymbol{E}} \tilde{\boldsymbol{R}}_3^{-1} \tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{V}}^\top \boldsymbol{\xi}_5(t) \end{aligned} \tag{21}$$

Then, from (4), we get

$$\mathbb{E}\Big\{(\tilde{\boldsymbol{B}}_{il} \tilde{\boldsymbol{\Gamma}} \tilde{\boldsymbol{K}}_j \tilde{\boldsymbol{x}}(t))^\top \tilde{\boldsymbol{R}} (\tilde{\boldsymbol{B}}_{il} \tilde{\boldsymbol{\Gamma}} \tilde{\boldsymbol{K}}_j \tilde{\boldsymbol{x}}(t))\Big\} = \tilde{\boldsymbol{x}}^\top(t) \Big\{\sum_{s=1}^{m} \pi_s^2 (\tilde{\boldsymbol{B}}_{il} \boldsymbol{\Sigma}_s \tilde{\boldsymbol{K}}_j)^\top \tilde{\boldsymbol{R}} \tilde{\boldsymbol{B}}_{il} \boldsymbol{\Sigma}_s \tilde{\boldsymbol{K}}_j\Big\} \tilde{\boldsymbol{x}}(t) \tag{22}$$

Let $\boldsymbol{\xi}(t) = \Big[\tilde{\boldsymbol{x}}^T(t) \quad \boldsymbol{x}^T(t-d(t)) \quad \boldsymbol{x}^T(t-\frac{d_1}{2}) \quad \boldsymbol{x}^T(t-d_1) \quad \boldsymbol{x}^T(t-\frac{d_2}{2}) \quad \boldsymbol{x}^T(t-d_2)\Big]^\top$. Combining (15)-(22), yields

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V(\tilde{\boldsymbol{x}}(t))\} \le & \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{l=1}^{r} \mu_i \hat{\nu}_j \hat{\mu}_l \boldsymbol{\xi}^T(t) \Big(\tilde{\boldsymbol{\psi}}_{11ijl}(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{A}}_{ijl}, \tilde{\boldsymbol{A}}_{di}) + \tilde{\boldsymbol{\psi}}_{ijl} \tilde{\boldsymbol{R}} \tilde{\boldsymbol{\psi}}_{ijl}^\top \\ & + \sum_{s=1}^{m} \pi_s^2 \boldsymbol{I}_\pi \otimes (\tilde{\boldsymbol{B}}_{il} \boldsymbol{\Sigma}_s \tilde{\boldsymbol{K}}_j)^\top \tilde{\boldsymbol{R}} (\boldsymbol{I}_\pi^\top \otimes (\tilde{\boldsymbol{B}}_{il} \boldsymbol{\Sigma}_s \tilde{\boldsymbol{K}}_j)\Big) \boldsymbol{\xi}(t) \end{aligned} \tag{23}$$

Hence, by performing the complement of Schur to (12), yields

$$\tilde{\boldsymbol{\psi}}_{11ijl}(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{A}}_{ijl}, \tilde{\boldsymbol{A}}_{di}) + \tilde{\boldsymbol{\psi}}_{ijl} \tilde{\boldsymbol{R}} \tilde{\boldsymbol{\psi}}_{ijl}^\top + \sum_{s=1}^{m} \pi_s^2 \boldsymbol{I}_\pi \otimes (\tilde{\boldsymbol{B}}_{il} \boldsymbol{\Sigma}_s \tilde{\boldsymbol{K}}_j)^\top \tilde{\boldsymbol{R}} (\boldsymbol{I}_\pi^\top \otimes (\tilde{\boldsymbol{B}}_{il} \boldsymbol{\Sigma}_s \tilde{\boldsymbol{K}}_j) < 0$$

Thus, it is obvious that $\mathbb{E}\{\mathcal{L}V(\tilde{\boldsymbol{x}}(t))\} < 0$, and system (11) is stochastically stable.
Next, we prove the regularity and impulse-free properties of system (11). From (12), we know $\tilde{\boldsymbol{\Psi}}_{11ijl} < 0$ which implies that

$$\text{sym}(\tilde{\boldsymbol{\Pi}}^\top \tilde{\boldsymbol{A}}_{ijl}) + \text{sym}(\tilde{\boldsymbol{M}}_1 \tilde{\boldsymbol{E}}) < 0 \tag{24}$$

For matrix $\tilde{\boldsymbol{E}}$, there exist $\mathbb{M}$ and $\mathbb{N}$, two non-singular matrices such that

$$\begin{aligned} &\mathbb{E} = \mathbb{M} \tilde{\boldsymbol{E}} \mathbb{N} = \begin{bmatrix} I_{2q} & 0 \\ 0 & 0 \end{bmatrix}, \qquad \hat{\boldsymbol{A}}_{ijl} = \mathbb{M} \tilde{\boldsymbol{A}}_{ijl} \mathbb{N} = \begin{bmatrix} \hat{\boldsymbol{A}}_{ijl11} & \hat{\boldsymbol{A}}_{ijl12} \\ \hat{\boldsymbol{A}}_{ijl21} & \hat{\boldsymbol{A}}_{ijl22} \end{bmatrix}, \quad \hat{\boldsymbol{\Pi}} = \mathbb{M}^{-T} \tilde{\boldsymbol{\Pi}} \mathbb{N} = \begin{bmatrix} \hat{\boldsymbol{\Pi}}_{11} & \hat{\boldsymbol{\Pi}}_{12} \\ \hat{\boldsymbol{\Pi}}_{21} & \hat{\boldsymbol{\Pi}}_{22} \end{bmatrix} \\ &\hat{\boldsymbol{M}}_1 = \mathbb{M}^{-T} \boldsymbol{M}_1 \mathbb{M}^{-1} = \begin{bmatrix} \hat{\boldsymbol{M}}_{111} & \hat{\boldsymbol{M}}_{112} \\ \hat{\boldsymbol{M}}_{121} & \hat{\boldsymbol{M}}_{122} \end{bmatrix} \end{aligned} \tag{25}$$

Based on lemma 2, we know that $\tilde{\boldsymbol{E}}^\top \tilde{\boldsymbol{\Pi}} = \tilde{\boldsymbol{\Pi}}^\top \tilde{\boldsymbol{E}}$ which implies that $\hat{\boldsymbol{\Pi}}_{12} = 0$, using (25).
Pre- and post-multiplying (24) by $\mathbb{N}^\top$ and $\mathbb{N}$, respectively, the inequality shown below holds

$$\text{sym}\,(\hat{\boldsymbol{\Pi}}_{22}^\top \hat{\boldsymbol{A}}_{ijl22}) < 0$$

which means that $\hat{\boldsymbol{A}}_{ijl22}$ is non-singular and system (11) is regular and impulse free, according to the definitions stated in [4].

□

3.2 Observer Design

As a result of the bilinear terms involved in condition (12), it is difficult to parameterize the controller and observer gains. In the sequel, we provide the procedure to synthesize the gains $\bar{\boldsymbol{K}}_i$, and $\bar{\boldsymbol{L}}_i$.

Theorem 4. *For given constants d_1, d_2 and $0 \le d_r < 1$, assume (A1) holds. System (11) is stochastically admissible, if there exist scalars α, β, λ_c, $c = 1,2,3,4$ and matrices $\bar{\boldsymbol{P}}_k > 0$, $\boldsymbol{J}_k > 0$, $\bar{\boldsymbol{M}}_k$, $\bar{\boldsymbol{N}}_k$, $\bar{\boldsymbol{T}}_k$ $\bar{\boldsymbol{V}}_k$, $\bar{\boldsymbol{X}}_k$, $k = 1,2$, $\bar{\boldsymbol{Q}} > 0$, $\bar{\boldsymbol{R}}_1 > 0$, $\bar{\boldsymbol{R}}_2 > 0$, $\bar{\boldsymbol{R}}_3 > 0$, $\bar{\boldsymbol{Q}}_a = \begin{bmatrix} \bar{\boldsymbol{Q}}_{11} & \bar{\boldsymbol{Q}}_{12} \\ \bar{\boldsymbol{Q}}_{12}^\top & \bar{\boldsymbol{Q}}_{22} \end{bmatrix} > 0$, $\bar{\boldsymbol{S}}_a = \begin{bmatrix} \bar{\boldsymbol{S}}_{11} & \bar{\boldsymbol{S}}_{12} \\ \bar{\boldsymbol{S}}_{12}^\top & \bar{\boldsymbol{S}}_{22} \end{bmatrix} > 0$, $\boldsymbol{\Lambda}$, $\bar{\boldsymbol{W}}_l$, $\boldsymbol{Y}_j \in \mathbb{R}^{m\times n}$, and $\boldsymbol{F}_l \in \mathbb{R}^{n\times n_y}$, the following LMIs hold under the condition $\hat{\mu}_l - \sigma_l \hat{\nu}_l \ge 0$ where σ_l is positive scalar for $i,j,l \in \mathbb{S}$:*

$$\begin{cases} \boldsymbol{\Xi}_{ijl} - \boldsymbol{\Lambda} < 0 \\ \sigma_j \boldsymbol{\Xi}_{ijj} - \sigma_j \boldsymbol{\Lambda} + \boldsymbol{\Lambda} < 0 \\ \sigma_l(\boldsymbol{\Xi}_{ijl} - \boldsymbol{\Lambda}) + \sigma_j(\boldsymbol{\Xi}_{ilj} - \boldsymbol{\Lambda}) + 2\boldsymbol{\Lambda} < 0, \quad l > j \end{cases} \tag{26}$$

where $\bar{\boldsymbol{\Pi}}_1 = \bar{\boldsymbol{P}}_1 \boldsymbol{E}^\top + \boldsymbol{U}^\top \bar{\boldsymbol{X}}_1 \boldsymbol{V}^\top$, $\bar{\boldsymbol{\Pi}}_2 = \bar{\boldsymbol{P}}_2 \boldsymbol{E}^\top + \boldsymbol{U}^\top \bar{\boldsymbol{X}}_2 \boldsymbol{V}^\top$, $\bar{\boldsymbol{\Pi}} = \operatorname{diag}\{\bar{\boldsymbol{\Pi}}_1, \bar{\boldsymbol{\Pi}}_2\}$, $\breve{\boldsymbol{\Pi}}_1 = \operatorname{diag}\{\bar{\boldsymbol{\Pi}}_1, \bar{\boldsymbol{\Pi}}_1\}$

$$\boldsymbol{\Xi}_{ijl} = \begin{bmatrix} \bar{\boldsymbol{\Xi}}_{ijl} & \alpha \boldsymbol{\Upsilon}_{1ij} & \boldsymbol{\Upsilon}_2 & \beta \boldsymbol{\Upsilon}_{3l} & \boldsymbol{\Upsilon}_{4l} \\ * & -\alpha \operatorname{sym}(\bar{\boldsymbol{\Pi}}_1) + \boldsymbol{J}_1 & 0 & 0 & 0 \\ * & * & -\boldsymbol{J}_1 & 0 & 0 \\ * & * & * & -\beta \operatorname{sym}(\bar{\boldsymbol{W}}_l) + \boldsymbol{J}_2 & 0 \\ * & * & * & * & -\boldsymbol{J}_2 \end{bmatrix} \tag{27}$$

$$\bar{\boldsymbol{\Xi}}_{ijl} = \begin{bmatrix} \bar{\boldsymbol{\Xi}}_{11ijl} & \sqrt{\frac{d_1}{2}}\bar{\boldsymbol{\psi}} & \sqrt{\frac{d_{12}}{2}}\bar{\boldsymbol{\psi}} & \sqrt{d_{12}}\bar{\boldsymbol{\psi}} & \bar{\boldsymbol{\psi}}_{15} & \boldsymbol{I}_\pi \otimes \bar{\boldsymbol{\psi}}_{16ij} \\ * & \bar{\boldsymbol{\psi}}_{22} & 0 & 0 & 0 & 0 \\ * & * & \bar{\boldsymbol{\psi}}_{33} & 0 & 0 & 0 \\ * & * & * & \bar{\boldsymbol{\psi}}_{44} & 0 & 0 \\ * & * & * & * & -\bar{\boldsymbol{\psi}}_{55} & 0 \\ * & * & * & * & * & -\bar{\boldsymbol{\psi}}_{66} \end{bmatrix}$$

$$\boldsymbol{\Upsilon}_{1ij} = \operatorname{col}\left\{\bar{\boldsymbol{B}}_{\Gamma ij} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \sqrt{\tfrac{d_1}{2}}\bar{\boldsymbol{B}}_{\Gamma ij} \quad \sqrt{\tfrac{d_{12}}{2}}\bar{\boldsymbol{B}}_{\Gamma ij} \quad \sqrt{d_{12}}\bar{\boldsymbol{B}}_{\Gamma ij} \quad 0 \quad \bar{\boldsymbol{B}}_{\Sigma ij}^\top\right\}$$

$$\boldsymbol{\Upsilon}_{3l} = \operatorname{col}\left\{\bar{\boldsymbol{F}}_l \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \sqrt{\tfrac{d_1}{2}}\bar{\boldsymbol{F}}_l \quad \sqrt{\tfrac{d_{12}}{2}}\bar{\boldsymbol{F}}_l \quad \sqrt{d_{12}}\bar{\boldsymbol{F}}_l \quad 0 \quad 0\right\}$$

$$\bar{\boldsymbol{\psi}} = \operatorname{col}\left\{\bar{\boldsymbol{A}}_{ijl}^\top \quad \bar{\boldsymbol{A}}_{di}^\top \quad 0 \quad 0 \quad 0 \quad 0\right\}$$

$$\bar{\boldsymbol{\psi}}_{151} = \sqrt{\frac{d_1}{2}} \operatorname{col}\left\{\bar{\boldsymbol{E}}^\top \bar{\boldsymbol{M}}_1 \quad 0 \quad \bar{\boldsymbol{E}}^\top \bar{\boldsymbol{M}}_2 \quad 0 \quad 0 \quad 0\right\}$$

$$\bar{\boldsymbol{\psi}}_{152} = \sqrt{\frac{d_{12}}{2}} \operatorname{col}\left\{0 \quad 0 \quad \bar{\boldsymbol{E}}^\top \bar{\boldsymbol{N}}_1 \quad 0 \quad \bar{\boldsymbol{E}}^\top \bar{\boldsymbol{N}}_2 \quad 0\right\},$$

$$\bar{\boldsymbol{\psi}}_{153} = \sqrt{d_{12}} \operatorname{col}\left\{0 \quad \bar{\boldsymbol{E}}^\top \bar{\boldsymbol{T}}_1 \quad 0 \quad \bar{\boldsymbol{E}}^\top \bar{\boldsymbol{T}}_2 \quad 0 \quad 0 \quad \right\}$$

$$\bar{\boldsymbol{\psi}}_{154} = \sqrt{d_{12}} \operatorname{col}\left\{0 \quad \bar{\boldsymbol{E}}^\top \bar{\boldsymbol{V}}_1 \quad \bar{\boldsymbol{E}}^\top \bar{\boldsymbol{V}}_2 \quad 0 \quad 0 \quad 0\right\}$$

$$\bar{\boldsymbol{B}}_{\Sigma ij} = \left[\pi_1(\bar{\boldsymbol{B}}_{\Sigma ij}^1)^\top, \ldots, \pi_m(\bar{\boldsymbol{B}}_{\Sigma ij}^m)^\top\right]$$

$$\bar{\boldsymbol{\psi}}_{16ij} = \left[\pi_1(\bar{\boldsymbol{B}}_{\Sigma ij}^1 \times [1 \ \ -1])^\top, \ldots, \pi_m(\bar{\boldsymbol{B}}_{\Sigma ij}^m \times [1 \ \ -1])^\top\right]$$

$$\boldsymbol{\Upsilon}_2 = \operatorname{col}\left\{\bar{\boldsymbol{\Pi}}_{12}^\top \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right\}$$

$$\boldsymbol{\Upsilon}_{4l} = \operatorname{col}\left\{\bar{\boldsymbol{C}}_{il}^\top \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\right\}$$

$$\bar{\boldsymbol{\psi}}_{22} = \lambda_1^2 \tilde{\boldsymbol{R}}_1 - \lambda_1 \operatorname{sym}(\bar{\boldsymbol{\Pi}})$$

$$\bar{\boldsymbol{\psi}}_{33} = \lambda_2^2 \tilde{\boldsymbol{R}}_2 - \lambda_2 \operatorname{sym}(\bar{\boldsymbol{\Pi}})$$

$$\bar{\boldsymbol{\psi}}_{44} = \lambda_3^2 \tilde{\boldsymbol{R}}_3 - \lambda_3 \operatorname{sym}(\bar{\boldsymbol{\Pi}})$$

$$\bar{\boldsymbol{\psi}}_{15} = \begin{bmatrix} \bar{\boldsymbol{\psi}}_{151} & \bar{\boldsymbol{\psi}}_{152} & \bar{\boldsymbol{\psi}}_{153} & \bar{\boldsymbol{\psi}}_{154} \end{bmatrix}$$

$$\bar{\boldsymbol{\psi}}_{55} = \operatorname{diag}\left\{\bar{\boldsymbol{R}}_1, \ \bar{\boldsymbol{R}}_2, \ \bar{\boldsymbol{R}}_3, \ \bar{\boldsymbol{R}}_3\right\}$$

$$\bar{\boldsymbol{\psi}}_{66} = \lambda_4^2 \tilde{\boldsymbol{R}} - \lambda_4 \operatorname{sym}(\breve{\boldsymbol{\Pi}}_1),$$

$$\bar{\Xi}_{11ijl} = \begin{bmatrix} \hat{\Xi}_{11ijl} & \hat{\Xi}_{12i} & \hat{\Xi}_{13i} & 0 & \bar{S}_{12} & 0 \\ * & \hat{\Xi}_{22i} & 0 & \hat{\Xi}_{24i} & 0 & \hat{\Xi}_{26i} \\ * & * & \hat{\Xi}_{33i} & -\bar{Q}_{12} & \hat{\Xi}_{35i} & 0 \\ * & * & * & \hat{\Xi}_{44i} & 0 & 0 \\ * & * & * & * & \hat{\Xi}_{55i} & -\bar{S}_{12} \\ * & * & * & * & * & \hat{\Xi}_{66i} \end{bmatrix}$$

$$\begin{aligned} \bar{\Xi}_{11ijl} &= \bar{Q}_{11} + \bar{S}_{11} + \bar{Q}_1 + \mathrm{sym}(\bar{M}_1\bar{E}^\top) + \mathrm{sym}(\bar{A}_{ijl}) & \bar{\Xi}_{26i} &= -\bar{T}_1\bar{E}^\top + (\bar{T}_2\bar{E}^\top)^\top \\ \bar{\Xi}_{12i} &= \bar{A}_{di} & \bar{\Xi}_{33i} &= \bar{Q}_{22} - \bar{Q}_{11} - \mathrm{sym}(\bar{M}_2\bar{E}^\top) + \mathrm{sym}(\bar{N}_1\bar{E}^\top) \\ \bar{\Xi}_{13i} &= \bar{Q}_{12} - \bar{M}_1\bar{E}^\top + (\bar{M}_2\bar{E}^\top)^\top & \bar{\Xi}_{35i} &= -\bar{N}_1\bar{E}^\top + \bar{N}_2\bar{E}^\top \\ \bar{\Xi}_{15i} &= -\bar{N}_1\bar{E}^\top + (\bar{N}_2\bar{E}^\top)^\top & \bar{\Xi}_{44i} &= -\bar{Q}_{22} + \mathrm{sym}(\bar{V}_1\bar{E}^\top) \\ \bar{\Xi}_{22i} &= -(1-h_d)\bar{Q}_1 + \mathrm{sym}(\bar{T}_1\bar{E}^\top) - \mathrm{sym}(\bar{V}_2\bar{E}^\top) & \bar{\Xi}_{55i} &= \bar{S}_{22} - \bar{S}_{11} - \mathrm{sym}(\bar{N}_2\bar{E}^\top) \\ \bar{\Xi}_{24i} &= \bar{V}_2\bar{E}^\top - (\bar{V}_1\bar{E}^\top)^\top & \bar{\Xi}_{66i} &= -\bar{S}_{22} - \mathrm{sym}(\bar{T}_2\bar{E}^\top) \end{aligned}$$

$$\bar{A}_{ijl} = \begin{bmatrix} A_i\bar{\Pi}_1 + B_i\bar{\Gamma}Y_j & -B_i\bar{\Gamma}Y_j \\ (A_i - A_l)\bar{\Pi}_1 + (B_i - B_l)\bar{\Gamma}Y_j - F_l(C_i - C_l) & A_i\bar{\Pi}_2 - (B_i - B_l)\bar{\Gamma}Y_j - F_lC_l \end{bmatrix}, \ \bar{A}_{di} = \begin{bmatrix} A_{di}\bar{\Pi}_1 & 0 \\ A_{di}\bar{\Pi}_1 & 0 \end{bmatrix}$$

$$\bar{B}_{\Gamma ij} = \begin{bmatrix} B_i\bar{\Gamma}Y_j \\ (B_i - B_l)\bar{\Gamma}Y_j \end{bmatrix}, \ \bar{B}^s_{\Sigma ij} = \begin{bmatrix} B_i\Sigma_sY_j \\ (B_i - B_l)\Sigma_sY_j \end{bmatrix}, \ \bar{\Pi}_{12} = \begin{bmatrix} 0 & \bar{\Pi}_1 - \bar{\Pi}_2 \end{bmatrix}, \ \bar{F}_l = \begin{bmatrix} 0 \\ F_l \end{bmatrix}$$

$$\bar{C}_{il} = \begin{bmatrix} \bar{W}_l(C_i - C_l) - (C_i - C_l)\bar{\Pi}_1 & \bar{W}_lC_i - C_i\bar{\Pi}_1 \end{bmatrix}$$

Moreover, parameters K_j and L_l are computed by $K_j = Y_j\bar{\Pi}_1^{-1}$ and $L_l = F_l\bar{W}_l^{-1}$, respectively.

Proof. In order to obtain less conservative results, the slack matrix provided below is used.

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\mu_i\hat{\nu}_j\Big(\hat{\nu}_l - \hat{\mu}_l\Big)\Lambda = 0 \tag{28}$$

where Λ is an arbitrary matrix with appropriate dimensions. Then, it can be known that

$$\begin{aligned} \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\mu_i\hat{\nu}_j\hat{\mu}_l\Xi_{ijl} &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\Big\{\mu_i\hat{\nu}_j\hat{\mu}_l\Xi_{ijl} + \mu_i\hat{\nu}_j\Big(\hat{\nu}_l - \hat{\mu}_l\Big)\Lambda\Big\} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\Big\{\mu_i\hat{\nu}_j(\hat{\mu}_l + \sigma_l\hat{\nu}_l - \sigma_l\hat{\nu}_l)\Xi_{ijl} + \mu_i\hat{\nu}_j\Big(\hat{\nu}_l - \hat{\mu}_l + \sigma_l\hat{\nu}_l - \sigma_l\hat{\nu}_l\Big)\Lambda\Big\} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\mu_i\hat{\nu}_j\Big\{\hat{\nu}_l\Big(\sigma_l\Xi_{ijl} - \sigma_l\Lambda + \Lambda\Big) + (\hat{\mu}_l - \sigma_l\hat{\nu}_l)\Big(\Xi_{ijl} - \Lambda\Big)\Big\} \\ &= \sum_{i=1}^{r}\mu_i\Bigg(\sum_{j=1}^{r}\hat{\nu}_j^2(\sigma_j\Xi_{ijj} - \sigma_j\Lambda + \Lambda) + \sum_{j=1}^{r-1}\sum_{l=j+1}^{r}\hat{\nu}_j\hat{\nu}_l(\sigma_j\Xi_{ijl} - \sigma_j\Lambda + \Lambda \\ &\quad + \sigma_l\Xi_{ilj} - \sigma_l\Lambda + \Lambda) \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r}\hat{\nu}_i(\hat{\mu}_l - \sigma_l\hat{\nu}_l)(\Xi_{ijl} - \Lambda)\Bigg) \end{aligned} \tag{29}$$

In light of the conditions in (26), the expression shown below holds.

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\mu_i\hat{\nu}_j\hat{\mu}_l\Xi_{ijl} < 0 \tag{30}$$

From $\Xi_{ijl} < 0$, we know that $-\lambda_1\,\mathrm{sym}(\bar{\Pi}) + \lambda_1^2\bar{R}_1 < 0$, and thus $\bar{\Pi}$ is non-singular.

Moreover, for $\bar{\boldsymbol{P}}_1 > 0$, it is easy to verify that $\boldsymbol{E}_L^\top \bar{\boldsymbol{P}}_1 \boldsymbol{E}_L > 0$. So according to Lemma 2 we have

$$\bar{\boldsymbol{\Pi}}_k^{-1} = (\boldsymbol{P}_k \boldsymbol{E} + \boldsymbol{U}^\top \boldsymbol{X}_k \boldsymbol{V}^\top) = \boldsymbol{\Pi}_k,\ k = 1,2 \quad \bar{\boldsymbol{\Pi}}^{-1} = \mathrm{diag}\,\{\boldsymbol{\Pi}_1, \boldsymbol{\Pi}_2\}$$

Using the fact that for any $\lambda_p > 0,\ p = 1,2,3$ the following condition holds:

$$0 \leq (\lambda_p \bar{\boldsymbol{\Pi}} - \tilde{\boldsymbol{R}}_p^{-1})^\top \tilde{\boldsymbol{R}}_p (\lambda_p \bar{\boldsymbol{\Pi}} - \tilde{\boldsymbol{R}}_p^{-1}) = \tilde{\boldsymbol{R}}_p^{-1} - \lambda_p \,\mathrm{sym}(\bar{\boldsymbol{\Pi}}) + \lambda_p^2 \bar{\boldsymbol{\Pi}}^\top \tilde{\boldsymbol{R}}_p \bar{\boldsymbol{\Pi}}$$

Then,

$$-\tilde{\boldsymbol{R}}_p^{-1} \leq -\lambda_p \,\mathrm{sym}(\bar{\boldsymbol{\Pi}}) + \lambda_p^2 \bar{\boldsymbol{\Pi}}^\top \tilde{\boldsymbol{R}}_p \bar{\boldsymbol{\Pi}}$$

Define $\mathbb{Z} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{I} \end{bmatrix}^\top$, and $\boldsymbol{\Gamma}_{ijl} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{\Upsilon}_{1ij} \bar{\boldsymbol{\Pi}}_1^{-1} & \boldsymbol{\Upsilon}_{3l} \bar{\boldsymbol{W}}_l^{-1} \end{bmatrix}$.

Multiplying $\mathbb{Z}^\top$ and its transposition in left and right sides to (30), respectively, to get

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} \mu_i \hat{\nu}_j \hat{\mu}_l \begin{bmatrix} \bar{\boldsymbol{\Xi}}_{ijl} & \alpha \boldsymbol{\Upsilon}_{1ij} + \boldsymbol{\Upsilon}_2 & \beta \boldsymbol{\Upsilon}_{3l} + \boldsymbol{\Upsilon}_{4il} \\ * & -\alpha \,\mathrm{sym}(\bar{\boldsymbol{\Pi}}_1) & 0 \\ * & * & -\beta \,\mathrm{sym}(\bar{\boldsymbol{W}}_l) \end{bmatrix} < 0 \tag{31}$$

Then, by congruence transformation by $\boldsymbol{\Gamma}_{ijl}$, we know that

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} \mu_i \hat{\nu}_j \hat{\mu}_l (\bar{\boldsymbol{\Xi}}_{ijl} + \mathrm{sym}\,(\boldsymbol{\Upsilon}_{1ij} \bar{\boldsymbol{\Pi}}_1^{-1} \boldsymbol{\Upsilon}_2^\top + \boldsymbol{\Upsilon}_{3l} \bar{\boldsymbol{W}}_l^{-1} \boldsymbol{\Upsilon}_{4il}^\top)) < 0 \tag{32}$$

Let $\boldsymbol{Y}_j = \boldsymbol{K}_j \bar{\boldsymbol{\Pi}}_1$ and $\bar{\boldsymbol{F}}_l = \boldsymbol{L}_l \bar{\boldsymbol{W}}_l$. From the terms $\boldsymbol{\Upsilon}_{1ij} \bar{\boldsymbol{\Pi}}_1^{-1} \boldsymbol{\Upsilon}_2^\top$, and $\boldsymbol{\Upsilon}_{3l} \bar{\boldsymbol{W}}_l^{-1} \boldsymbol{\Upsilon}_{4il}^\top$ one can calculate

$$\bar{\boldsymbol{F}}_l \bar{\boldsymbol{W}}_l^{-1} \bar{\boldsymbol{C}}_{il} = \begin{bmatrix} 0 & 0 \\ \bar{\boldsymbol{F}}_l (\boldsymbol{C}_i - \boldsymbol{C}_l) - \boldsymbol{L}_l (\boldsymbol{C}_i - \boldsymbol{C}_l) \bar{\boldsymbol{\Pi}}_1 & \bar{\boldsymbol{F}}_l \boldsymbol{C}_i - \boldsymbol{L}_l \boldsymbol{C}_i \bar{\boldsymbol{\Pi}}_2 \end{bmatrix},$$

$$\bar{\boldsymbol{B}}_{\Gamma ij} \bar{\boldsymbol{\Pi}}_1^{-1} \bar{\boldsymbol{\Pi}}_{12} = \begin{bmatrix} 0 & \boldsymbol{B}_i \bar{\boldsymbol{\Gamma}} \boldsymbol{Y}_j - \boldsymbol{B}_i \bar{\boldsymbol{\Gamma}} \boldsymbol{K}_j \Pi_2 \\ 0 & (\boldsymbol{B}_i - \boldsymbol{B}_l) \bar{\boldsymbol{\Gamma}} \boldsymbol{Y}_j - (\boldsymbol{B}_i - \boldsymbol{B}_l) \bar{\boldsymbol{\Gamma}} \boldsymbol{K}_j \Pi_2 \end{bmatrix}$$

$$\bar{\boldsymbol{B}}_{\Sigma ij}^s \bar{\boldsymbol{\Pi}}_1^{-1} \bar{\boldsymbol{\Pi}}_{12} = \begin{bmatrix} 0 & \boldsymbol{B}_i \boldsymbol{\Sigma}_s \boldsymbol{Y}_j - \boldsymbol{B}_i \boldsymbol{\Sigma}_s \boldsymbol{K}_j \Pi_2 \\ 0 & (\boldsymbol{B}_i - \boldsymbol{B}_l) \boldsymbol{\Sigma}_s \boldsymbol{Y}_j - (\boldsymbol{B}_i - \boldsymbol{B}_l) \boldsymbol{\Sigma}_s \boldsymbol{K}_j \Pi_2 \end{bmatrix}, s = 1,2,\cdots m$$

Thus, it can be concluded that

$$\bar{\boldsymbol{A}}_{ijl} + \bar{\boldsymbol{B}}_{\Gamma ij} \bar{\boldsymbol{\Pi}}_1^{-1} \bar{\boldsymbol{\Pi}}_{12} + \bar{\boldsymbol{F}}_l \bar{\boldsymbol{W}}_l^{-1} \bar{\boldsymbol{C}}_{il} = \tilde{\boldsymbol{A}}_{ijl} \bar{\boldsymbol{\Pi}}, \quad \bar{\boldsymbol{B}}_{\Sigma ij}^s \times [1 \ \ -1] + \bar{\boldsymbol{B}}_{\Sigma ij}^s \bar{\boldsymbol{\Pi}}_1^{-1} \bar{\boldsymbol{\Pi}}_{12} = \tilde{\boldsymbol{B}}_{il} \boldsymbol{\Sigma}_s \tilde{\boldsymbol{K}}_j \bar{\boldsymbol{\Pi}} \tag{33}$$

Using (33), and performing the congruence transformation to (32) by $diag(\boldsymbol{\Pi}, \boldsymbol{\Pi}, \boldsymbol{\Pi}, \boldsymbol{\Pi}, \boldsymbol{\Pi}, \boldsymbol{\Pi}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I})$ and its transpose

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} \mu_i \hat{\nu}_j \hat{\mu}_l \tilde{\boldsymbol{\Psi}}_{ijl}(\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{A}}_{ijl}, \tilde{\boldsymbol{A}}_{di}) < 0$$

holds, using the following expressions:

$$\tilde{\boldsymbol{Q}}_{uv} = \bar{\boldsymbol{\Pi}}^{-T} \bar{\boldsymbol{Q}}_{uv} \bar{\boldsymbol{\Pi}}^{-1}, \quad \tilde{\boldsymbol{S}}_{uv} = \bar{\boldsymbol{\Pi}}^{-T} \bar{\boldsymbol{S}}_{uv} \bar{\boldsymbol{\Pi}}^{-1}, \quad \tilde{\boldsymbol{W}}_k = \bar{\boldsymbol{\Pi}}^{-T} \bar{\boldsymbol{W}}_k \bar{\boldsymbol{\Pi}}^{-1},\ \tilde{\boldsymbol{M}}_k = \bar{\boldsymbol{\Pi}}^{-T} \bar{\boldsymbol{M}}_k \bar{\boldsymbol{\Pi}}^{-1},$$
$$\tilde{\boldsymbol{V}}_k = \bar{\boldsymbol{\Pi}}^{-T} \bar{\boldsymbol{V}}_k \bar{\boldsymbol{\Pi}}^{-1}, \quad \tilde{\boldsymbol{T}}_k = \bar{\boldsymbol{\Pi}}^{-T} \bar{\boldsymbol{T}}_k \bar{\boldsymbol{\Pi}}^{-1}, \quad \tilde{\boldsymbol{R}}_c = \bar{\boldsymbol{\Pi}}^{-1} \bar{\boldsymbol{R}}_c \bar{\boldsymbol{\Pi}}^{-T},\ (u,v,k = 1,2,\ c = 1,2,3),$$

Hence, according to Theorem 3, closed-loop system (11) is stochastically admissible.

□

Remark 5. *Here, we address the issue of mismatched property between $\mu_i(\boldsymbol{x}(t))$ and $\nu_i(\hat{\boldsymbol{x}}(t))$. By introducing a slack matrix $\boldsymbol{\Lambda}$ to express (28) which is combined with the condition $\hat{\mu}_l - \sigma_l \hat{\nu}_l \geq 0$ to get relaxed conditions as stated in (26). This technique incorporates membership function information into stability conditions, leading to less conservative conditions.*

Remark 6. *Our approach for designing the controller and observer gains relies on decoupling the BMI terms by using slack matrices as defined in (33). Comparatively, this approach differs from that presented in [2], in which the Finsler lemma is investigated and auxiliary variables T_i are employed to validate $C_i T_i = [1 \ \ 0]$ and reach the desired gains.*

4 Numerical examples

In this section, we provide a concise description of the computational framework and present two examples to illustrate the effectiveness and advantages of the proposed control scheme.

4.1 Computational Framework and Algorithm

Computing experiments were performed using the Matlab programming language and a computer with the following characteristics:(i) [OS] Windows 10 Enterprise for 64 bits; (ii) [RAM] 8 Gigabytes; and (iii) [Processor] Intel(R) Core(TM) i7-4790T CPU @ 2.70 GigaHertz.
A detailed explanation of the design procedure is given in Algorithm 1 and the flowchart displayed in Figure 1. Using `Yalmip` software and the optimization toolbox `mosek`, the algorithm 1 was executed.

Algorithm 1 Procedure design

1: Describe the non-linear system by the IVF system.
2: Choose the parameters of the actuator fault model described in (3).
3: Determine the gains $\boldsymbol{K}_j$, $\boldsymbol{L}_l$ of the LMI established in Theorem 4.
4: Design the observer indicated in (7)
5: Apply the designed control law expressed in (9) to the model.

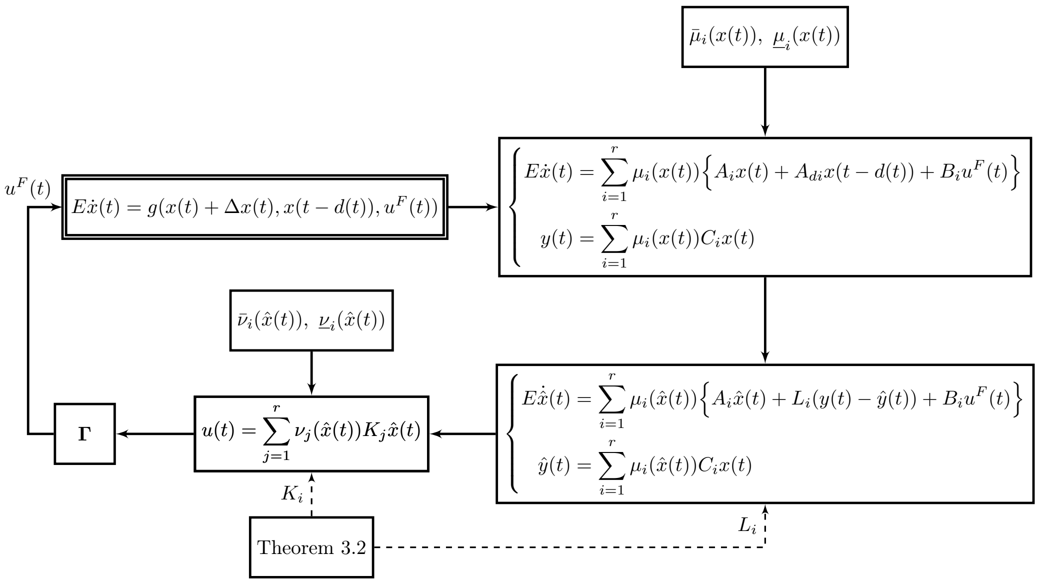


Figure 1: Flowchart of the control procedure.

The flowchart displayed in Figure 1 provides a clear description of the proposed design procedure.

4.2 Example 1

The efficiency and correctness of the suggested control scheme is illustrated with the example provided from [30] with the following matrices:

$$\boldsymbol{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ 0 & -1 & -0.6 \end{bmatrix} \quad \boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0.6 \end{bmatrix} \quad \boldsymbol{A}_{d1} = \boldsymbol{A}_{d2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.13 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{B}_1 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{B}_2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad \boldsymbol{C}_{21} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \qquad \boldsymbol{C}_{22} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

The lower and upper bounds of membership functions of the corresponding interval-valued TS fuzzy model are listed in Table 5.

Lower membership functions	Upper membership functions
$\underline{\mu}_1(x_2) = 0.95 - \dfrac{0.95}{1+\exp\left(\frac{x_2+4.5}{8}\right)}$ $\underline{\mu}_2(x_2) = 1 - \underline{\mu}_1(x_2)$	$\bar{\mu}_1(x_2) = 0.95 - \dfrac{0.925}{1+\left(\frac{x_2+3.5}{8}\right)}$ $\bar{\mu}_2(x_2) = 1 - \bar{\mu}_2(x_2)$

Table 2: Lower and upper membership functions of the plant.

The weighting functions are chosen as $\underline{\alpha}_i = sin^2(x_2(t))$ and $\bar{\alpha}_i = 1 - \underline{\alpha}_i$ for $i = 1, 2$. For the aforementioned interval type-2 fuzzy singular system, we are going to design an observer-based IVF controller that makes a closed-loop system stochastically admissible. Accordingly, we have defined the lower and upper bounds of membership functions for interval-valued fuzzy controllers in Table 6. To determine what constitutes membership functions, non-linear weight functions are selected as $\underline{\beta}_j(x_2(t)) = \cos^2(x_2(t))$ and $\bar{\beta}_j(x_1(t)) = 1 - \underline{\beta}_j(x_2(t))$.

Lower membership functions	Upper membership functions
$\underline{\nu}_1(x_2) = 1 - \dfrac{1}{1+\exp\left(\frac{x_2+5}{2}\right)}$ $\underline{\nu}_2(x_2) = 1 - \underline{\nu}_1(x_2)$	$\bar{\nu}_1(x_2) = 1 - \dfrac{1}{1+\exp\left(\frac{x_2+4}{2}\right)}$ $\bar{\nu}_2(x_2) = 1 - \bar{\nu}_1(x_2)$

Table 3: Lower and upper membership functions of the controller.

In this example, it is assumed that $d(t) = 0.2 + 0.1\sin(t)$, which provides that $d_1 = 0.1$, $d_2 = 0.3$, $d_r = 0.1$.
Define

$$\boldsymbol{E}_L = \boldsymbol{E}_R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad \boldsymbol{U} = \boldsymbol{V}^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Let $\sigma_1 = 0.1$, $\sigma_2 = 0.1$, $\alpha = 0.01$, $\beta = 0.002$, $\lambda_1 = 0.1$, $\lambda_2 = 5$, $\lambda_3 = 12$, and $\lambda_4 = 0.001$.
To demonstrate the efficacy of the proposed control strategy, two cases are considered using the previous values.

case 1 The actuator faults are assumed to occur randomly with a Bernoulli-distribution satisfying with $\bar{\boldsymbol{\Gamma}} = \text{diag}\{0.6, 0.55, 0.65\}$.

case 2 For this case, the controller is designed for a system without actuator faults, i.e. we take $\bar{\boldsymbol{\Gamma}} = \text{diag}\{1, 1, 1\}$.

According to the algorithm above, the LMIs (26) in Theorem 4, are solved for both cases with parameters displayed in table 4.

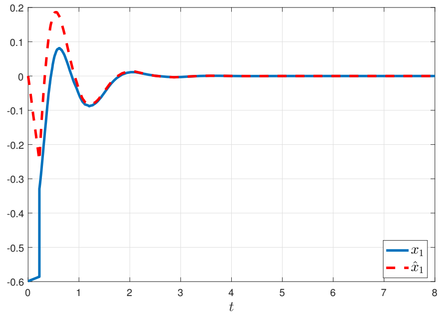
Case	parameters
Case 1	$\boldsymbol{K}_1 = \begin{bmatrix} -31.437 & -12.916 & 21.486 \\ -30.49 & -12.922 & 20.022 \\ 24.519 & 12.621 & -25.486 \end{bmatrix}$, $\boldsymbol{K}_2 = \begin{bmatrix} -31.594 & -12.971 & 21.314 \\ -30.656 & -12.977 & 19.833 \\ 24.7 & 12.686 & -25.253 \end{bmatrix}$, $\boldsymbol{L}_1 = \begin{bmatrix} 5.3154 & 2.0863 \\ 5.1943 & 0.2834 \\ -0.92299 & 1.5574 \end{bmatrix}$, $\boldsymbol{L}_2 = \begin{bmatrix} 4.1972 & 2.1951 \\ 5.1622 & 0.033923 \\ -0.44853 & 0.36828 \end{bmatrix}$
Case 2	$\boldsymbol{K}_1 = \begin{bmatrix} -11.896 & -6.8293 & -0.20661 \\ -11.698 & -7.0008 & -1.3811 \\ 11.704 & 7.7511 & -2.5207 \end{bmatrix}$, $\boldsymbol{K}_2 = \begin{bmatrix} -11.932 & -6.8332 & -0.20605 \\ -11.727 & -7.0052 & -1.3798 \\ 11.741 & 7.755 & -2.5222 \end{bmatrix}$, $\boldsymbol{L}_1 = \begin{bmatrix} 4.5107 & 1.5844 \\ 6.7738 & -3.0472 \\ -1.3806 & 1.9677 \end{bmatrix}$, $\boldsymbol{L}_2 = \begin{bmatrix} 3.5379 & 2.3054 \\ 8.0944 & -3.6425 \\ -0.94405 & 0.57315 \end{bmatrix}$

Table 4: Controller and observer gains for both cases

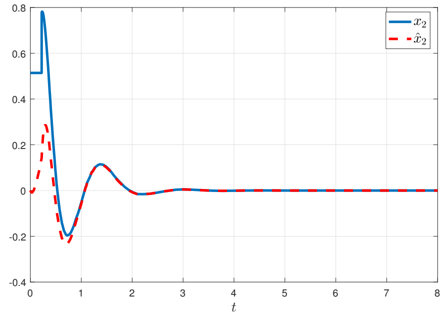
Three scenarios are considered in the numerical simulations:

1. The reliable observer-based controller obtained in the first case is applied to a healthy system;
2. The same observer-based controller designed in the first case is performed to a system suffering from actuator failure;
3. A system with an actuator failure is controlled by the unreliable observer-based controller developed in the second case.

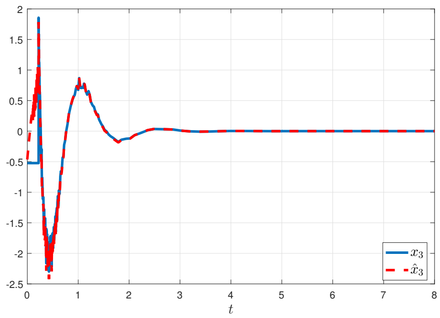
For all cases, the simulations are undergone with initial condition $\boldsymbol{\phi}(t) = [-0.73140,\ 0.5140,\ -0.5237]^\top$, $t \in [-0.3\ 0]$, and the results are depicted in Figures 2-4, from where the inputs as well as system and its related observer state variables are displayed. We observe that, when the reliable control law is applied the system dynamics are stabilized despite uncertainties and stochastic actuator failures, however, when the unreliable control law is applied to the system with failures, we see that the performance of the system is degraded. Thus, The simulations validate that the proposed control scheme is effective in accommodating actuator faults in the system and shows its robustness in spite of uncertainty.



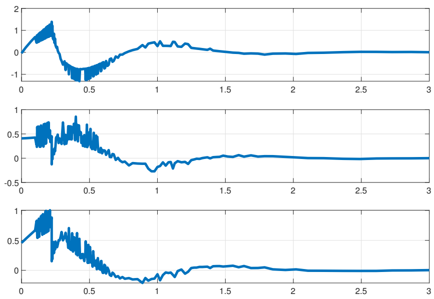
(a) The curves of $x_1(t)$, and $\hat{x}_1(t)$



(b) The curves of $x_2(t)$, and $\hat{x}_2(t)$

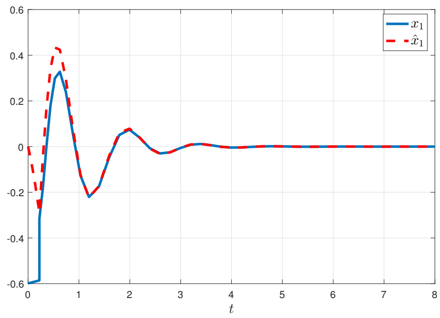


(c) Trajectories of $x_3(t)$, and $\hat{x}_3(t)$

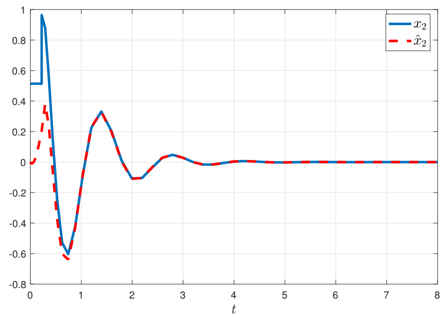


(d) Input trajectories

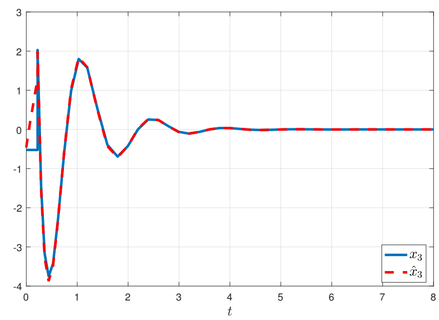
Figure 2: Simulation plots for failure mode



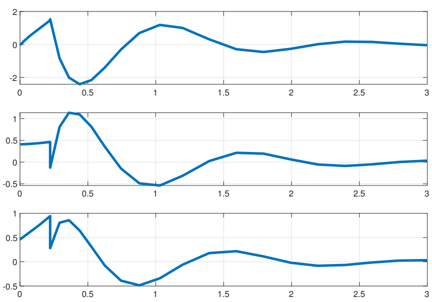
(a) The curves of $x_1(t)$, and $\hat{x}_1(t)$



(b) The curves of $x_2(t)$, and $\hat{x}_2(t)$

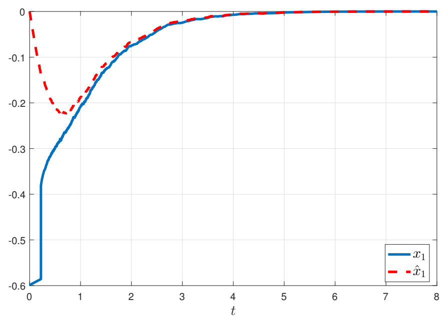


(c) Trajectories of $x_3(t)$, and $\hat{x}_3(t)$

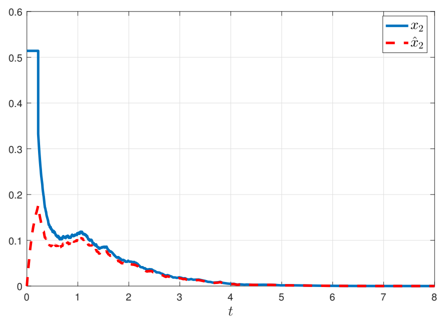


(d) Input trajectories

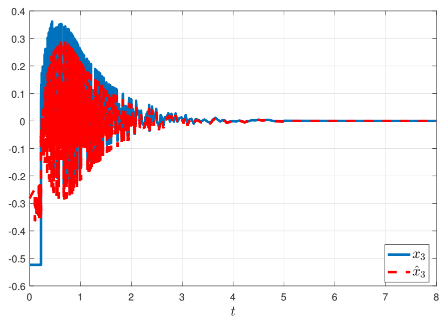
Figure 3: Simulation plots for normal mode



(a) The curves of $x_1(t)$, and $\hat{x}_1(t)$

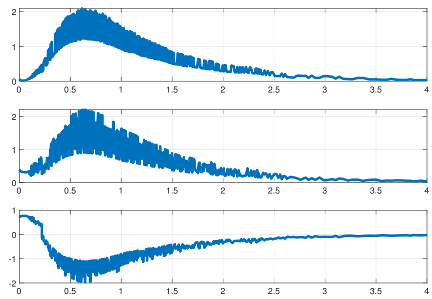


(b) The curves of $x_2(t)$, and $\hat{x}_2(t)$



(c) Trajectories of $x_3(t)$, and $\hat{x}_3(t)$

exam



(d) Input trajectories

Figure 4: Simulation plots for non-reliable controller gains in case 2

4.3 Bio-economic System

Using a bio-economic system adapted from [19], the proposed control scheme can be shown to be both efficient and correct. The bio-economic system is stated as

$$\begin{cases} \dot{z}_1(t) &= -0.5z_1(t) + 0.15z_2(t) - 0.01z_1^2(t) - E(t)z_1(t) + u_1(t), \\ \dot{z}_2(t) &= 0.5z_1(t) - 0.1z_2(t), \\ 0 &= E(t)(z_1(t) - 50) + u_2(t), \end{cases} \tag{34}$$

with $z_1(t)$ and $z_2(t)$ being the population density of, respectively, immature and mature species at time t. $E(t)$ corresponds to the harvest effort on the immature population, $u_1(t)$ and $u_2(t)$ state, respectively, the capture of an immature population and the government regulation, by means of a tax or subsidy, of a biological resource. Based on model (34), the following model is obtained by translating the positive equilibriums to zero [19] :

$$\begin{cases} \dot{x}_1(t) &= -1.25x_1(t) + 0.15x_2(t) - 50x_3(t) - 0.01x_1^2(t) - x_1(t)x_3(t) + u_1(t), \\ \dot{x}_2(t) &= 0.5x_1(t) - 0.1x_2(t), \\ 0 &= -0.75x_1(t) + x_1(t)x_3(t) + u_2(t), \end{cases} \tag{35}$$

Assume that $x_1(t) \in [-10\ 10]$, and $0 \leq \Delta x_1(t) \leq 2$. Given the uncertainty associated with the parameter $\Delta x_1(t)$, it is evident that the IT-2 T-S fuzzy system should be adopted to model nonlinear system (35). The lower and upper bounds of membership functions of the corresponding IT-2 TS fuzzy model are listed in Table 5.

Lower membership functions	Upper membership functions
$\underline{\mu}_1 = \dfrac{12 - x_1(t)}{22}$	$\bar{\mu}_1 = \dfrac{12 - (x_1(t) + 2)}{22}$
$\underline{\mu}_2 = \dfrac{10 + x_1(t)}{22}$	$\bar{\mu}_2 = \dfrac{10 + (x_1(t) + 2)}{22}$

Table 5: Lower and upper membership functions of the plant.

The weighting functions are chosen as $\underline{\alpha}_i = sin^2(x_2(t))$ and $\bar{\alpha}_i = 1 - \underline{\alpha}_i$ for $i = 1, 2$.
The interval-valued fuzzy system (1) is defined by $\boldsymbol{A}_i = c\bar{\boldsymbol{A}}_i$, $\boldsymbol{A}_{di} = (1-c)\bar{\boldsymbol{A}}_i$ and the following matrices:

$$\boldsymbol{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{\boldsymbol{A}}_1 = \begin{bmatrix} -1.15 & 0.15 & -40 \\ 0.5 & -0.1 & 0 \\ -0.75 & 0 & -10 \end{bmatrix}, \quad \bar{\boldsymbol{A}}_2 = \begin{bmatrix} -1.37 & 0.15 & -62 \\ 0.5 & -0.1 & 0 \\ -0.75 & 0 & 12 \end{bmatrix}, \quad \boldsymbol{B}_1 = \boldsymbol{B}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\boldsymbol{C}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad \boldsymbol{C}_{22} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

This example aims to design a fuzzy controller (9) that guarantees the admissibility of closed-loop systems. To accomplish this goal, table 6 lists the lower and upper bounds of membership functions to characterize the interval-valued fuzzy controller.

Lower membership functions	Upper membership functions
$\underline{\nu}_1(x_1) = 0.3e^{-(x_1^2/0.35)}$	$\bar{\nu}_1(x_1) = \underline{\nu}_1(x_1)$
$\underline{\nu}_2(x_1) = 1 - 0.3e^{-(x_1^2/0.35)}$	$\bar{\nu}_2(x_1) = \underline{\nu}_2(x_1)$

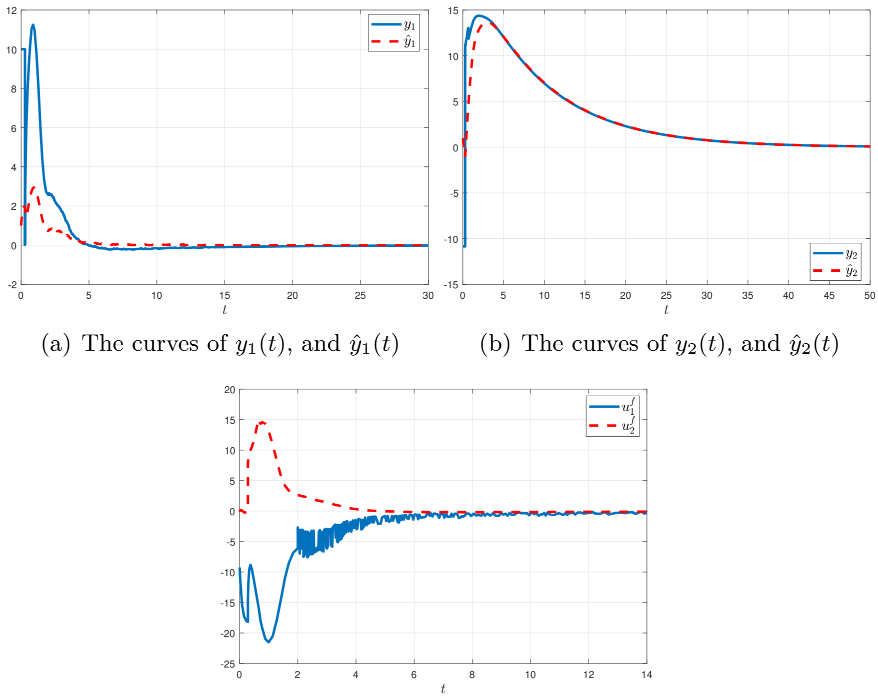
Table 6: Lower and upper membership functions of the controller.

To well determine the membership functions, the non-linear weight functions are selected as $\underline{\beta}_j(x_2(t)) = \cos^2(x_2(t))$ and $\bar{\beta}_j(x_2(t)) = 1 - \underline{\beta}_j(x_2(t))$.

When dealing with the reliable observer-based feedback control problem for the bio-economic system under a stochastic fault, we can solve the LMIs (26) in Theorem 4, with $c = 0.98$, $d_1 = 0.2$, $d_2 = 0.5$, $d_r = 0.3$, $\lambda_1 = 3$, $\lambda_2 = 11$, $\lambda_3 = 3$, $\lambda_4 = 11$, and $\bar{\boldsymbol{\Gamma}} = \text{diag}\{0.5,\ 0.6\}$, to obtain a feasible solution that involves the following corresponding observer and controller gains:

$$\begin{aligned} \boldsymbol{K}_1 &= \begin{bmatrix} -8.9716 & -0.058112 & -224 \\ 0.24388 & -0.0037578 & -973.93 \end{bmatrix}, \quad \boldsymbol{K}_2 = \begin{bmatrix} -9.0421 & -0.05894 & -434.76 \\ 0.23901 & -0.0035811 & -982.23 \end{bmatrix}, \\ \boldsymbol{L}_1 &= \begin{bmatrix} 1.9782 & -0.31557 \\ 0.48519 & 0.84128 \\ -0.42174 & 1.4254 \end{bmatrix}, \quad \boldsymbol{L}_2 = \begin{bmatrix} 3.1661 & 0.25049 \\ 0.44715 & 1.317 \\ -0.85805 & -2.3441 \end{bmatrix} \end{aligned} \tag{36}$$

For initial condition $\boldsymbol{\phi}(t) = [10,\ 0.5,\ 12]^\top$, $t \in [-0.5\ 0]$, and $d(t) = 0.2 + 0.3|\sin(t)|$, figure 5 shows the simulation results of the resulting closed-loop system, which was achieved by applying the developed fault-tolerant controller (36) to an uncertain system (34). This figure records the output responses of the system, and the control input $u^f(t)$. It can be concluded that, under actuator faults, uncertainties, and unmeasured states, the suggested controller would retain the closed-loop system dynamically stable.



(a) The curves of $y_1(t)$, and $\hat{y}_1(t)$ (b) The curves of $y_2(t)$, and $\hat{y}_2(t)$

(c) Trajectories of $u_1^f(t)$, and $u_2^f(t)$

Figure 5: Simulation plots for bio-economic system

5 Comparative Explanations

For non-linear singular systems exhibiting uncertain states with time varying delay as well as stochastic actuator failures, this paper explores the fault-tolerant observer-based control issue. The advantages over existing methods are listed below.

1. In comparison with existing findings [14, 20, 28, 37, 41], the outcomes developed in this paper are more realistic and general since the fuzzy model incorporates the intrinsic uncertainty of the system. Moreover, fuzzy systems and fuzzy controllers have different premise membership functions.

2. Despite having studied the observer-based control problem for interval-valued fuzzy systems [8, 48, 51], the present investigation differed with the following features:

   - our approach is more general since the system under consideration is a singular one, in which $E \neq I$,
   - typically, stochastic actuator failure is considered to cope with the reliable control problem for this class of systems.

3. Compared to the fuzzy static and dynamic controllers proposed in [6, 30], the observer-based control strategy developed in this study assumes that the premise variables are unmeasurable as suggested in [51]

6 Conclusion

An attempt is made in this study to deliver solutions to the main challenges that come up when dealing with non-linear singular systems, such as uncertainty, time-varying delay, and stochastic actuator failures. Based on an IVF model that exploits both the lower and upper membership functions to adequately characterize the uncertainties, an IVF observer is designed to estimate the unmeasured states, and then an IVF controller is synthesized to stabilize the system under consideration. The analysis of the existing of such observer-based controller is carried out by involving an appropriate

Lyapunov-Krasovskii functional, and the key point of the conceived control scheme lies in the use of decoupling matrix technique to establish a set of feasible LMI-based constraints so that the closed-loop system is stochastically admissible. The proposed control scheme has been validated by two numerical simulations. As part of prospective research topics, we will extend the suggested developments to non-linear systems with event-triggered output feedback control problems.

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Author contributions

The authors contributed equally to this work.

Conflict of interest

The authors declare no conflict of interest.

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