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Enhanced Solutions to Intuitionistic Fuzzy Multiobjective Linear Fractional Optimization Problems via Lexicographic Method

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Abstract

Optimization involving fuzzy numbers generally, and intuitionistic fuzzy numbers particularly is more and more an essential part of any computational intelligence method; and might be of interest in modeling fuzzy control systems, or carrying out a fuzzy sensitivity analysis. The recent literature includes many research papers related to both theoretical modelling, and practical implementation of fuzzy decision support systems. Fuzzy optimization is one of the main part in such decision-making tools.

A realistic solution to a fuzzy optimization problem is always desired, and similarly, a Pareto optimal solution to a multiple objective optimization problem is always a need. In this paper we analyze the shortcomings; and eliminate the weaknesses of a solution approach from the literature proposed by Moges et al. in 2023. Firstly, Moges et al. used an accuracy function to de-fuzzify the original intuitionistic fuzzy optimization problem; then linearized the obtained crisp problem; and finally involved fuzzy goals to solve the derived multiple objective linear problem.

We present two faulty key points of their approach, namely the de-fuzzification and linearization steps; prove the inappropriateness of their results; and propose some improvements. The final aim in addressing the first key point is to clarify the border between defuzzifications made in accordance to true theoretical statements, and those made for modeling reasons. The methodological improvement we propose is related to the second key point and it assures that Pareto optimal solutions - highly required in multiple objective optimization - are obtained. To illustrate our point of view, we use a numerical example from the literature, and report better numerical results derived by the improved methodology.

Keywords: fuzzy mathematical programming, linear fractional programming, multi-objective optimization, intuitionistic fuzzy numbers.

1 Introduction

Optimization involving intuitionistic fuzzy numbers is an essential part of many computational intelligence methods. Such optimization problems can be of interest in modelling fuzzy control systems, or carrying out a fuzzy sensitivity analysis. On the other side, the nowadays decision-making processes become more and more complex, since very often conflicting objectives have to be simultaneously optimized, and the optimization models have to incorporate uncertain information. Finding realistic solutions to intuitionistic fuzzy multiple objective optimization problems is aiming by many researches.

In the recent literature, one can find many research papers related to both theoretical modelling, and practical implementation of fuzzy decision support systems. Fuzzy optimization is one of the main part in such decision-making tools. A general view on the evolution from classic to fuzzy logic was presented in [15]. Sharma et al. [21] proposed an approach to solving multiple objective bi-level chance constrained optimization problem in an intuitionistic fuzzy environment. They modelled the uncertainty within the constraints system of the optimization problem using stochastic programming approaches. An equivalent crisp problem was constructed, and then solved using a TOPSIS method.

Figueroa-Garcia et al. [8] reviewed the literature and analysed the new trends in the field of fuzzy linear programming. They concluded that majority of the reviewed papers focused on solving de-fuzzified variants of the original problems, but there is a clear trend to explore new uncertainty representations. Valipour and Yaghoobi [25] reviewed the fuzzy linearization approaches used in the literature to solve multiple objective linear-fractional programming problems. They discussed the imposibility of finding equivalent linear multiple objective programming problems to linear-fractional ones even when the mapping between them is continuous.

Rani et al. [19] introduced the intuitionistic fuzzy numbers with non-linear grade functions. They generalized the classic techniques for solving intuitionistic fuzzy non-linear multiple objective optimization problems. Fuzzy sets [26], and later on intuitionistic fuzzy sets [2] were introduced as tools to handle the uncertainty. These concepts together with the theory developed around them have found a wide specter of applications [7]. They were easily and immediately adopted in the optimization and decision making framework to model uncertainty, perform arithmetic operations on uncertain data, formulate fuzzy goals and so on. Khan et. al [10] drew a parallel between the basic definitions of fuzzy, intuitionistic fuzzy and neutrosophic sets and norms, trying to disclose their meaning and establish a link between them and the classic sets and norms.

Many times, when dealing with fuzzy concepts relying on a simple analogy with crisp numbers can be misleading. The well known fuzzy arithmetic is widely used due to its power and simplicity, but still it can also produce meaningless results if it is not applied in a proper manner. Sotoudeh-Anvari [22] reviewed the main drawbacks and mathematical incorrect assumptions from the literature (published between 2010 and 2020) on which many fuzzy operations research methodologies rely.

There are many ways to order the intuitionistic fuzzy numbers. Recently, Popa [17] introduced a new method that ranks the trapezoidal intuitionistic fuzzy numbers by incorporating a parameter able to describe the attitude of the decision factors. Further on, the new ranking method was applied to multiple criteria decision making. De and Nandi [5] discussed a new, more accurate, method for defuzzification and/or computation of ranking indexes of various types of fuzzy sets.

Recently, Rizk-Allah et al. used intuitionistic fuzzy sets for non-linear multiple objective problems [20]; Nayak and Ojha [16] approached multipe objective non-linear fractional problems via fuzzy goals and TOPSIS concept; Lin et. al [11] involved trapezoidal intuitionistic fuzzy information in multi-attribute decision-making. The uncertainty was modeled based on the risk orientation and similarity measure. A description of applied fuzzy fractional programming can be found in [12] to minimal cost flow problems; and in [13] to education systems. Optimization involving fuzzy numbers might be of interest in modeling fuzzy control systems (see for instance [18], where optimization problems were defined to tune the parameters of the learning functions); or fuzzy sensitivity analysis (see Guerra et. al [9] who applied it to project financing transactions).

In this paper we analyze the shortcomings of the solution approach to intuitionistic fuzzy multiple objective linear fractional programming (IFMOLFP) problem introduced in [14]. We proceed by identifying the incorrectness in the proof of the theorem that stated the equivalence between the original IFMOLFP problem and the constructed crisp multiple objective linear fractional programming

(MOLFP) problem. Then, we refer the reader to the results presented in [24] showing that MOLFP problems cannot be equivalently linearized using the transformation proposed in [3] (that was intended to be an extension to the well known Charnes-Cooper transformation [4] of a single objective linear fractional programming (LFP) problem to a linear programming (LP) problem). We also propose theoretical improvements to the existing solution approach assuring that a Pareto optimal solution is obtained to the crisp MOLFP problem.

The remaining of the paper is structured as follows: after some preliminaries included in Section 2, we describe and prove our theoretical findings, and propose methodological improvements in Section 3. In Section 4, we use a numerical example from the literature to illustrate our point of view, and derive better solutions. Final conclusion and directions for further researches are presented Section 5.

2 Preliminaries

In this section we provide the notation and terminology needed in the sequel. They are related to the intuitionistic fuzzy sets, accuracy function, crisp and intuitionistic fuzzy multiple objective linear fractional programming problems.

2.1 The intuitionistic fuzzy sets

$\widetilde{A}^I$ is an intuitionistic fuzzy subset of a the universe X defined as a collection of triples

$$\left(x, \mu_{\widetilde{A}^I}(x), \nu_{\widetilde{A}^I}(x)\right)$$

such that $x \in X$, $\mu_{\widetilde{A}^I}(x)$, $\nu_{\widetilde{A}^I}(x) \in [0,1]$, and $0 \leq \mu_{\widetilde{A}^I}(x) + \nu_{\widetilde{A}^I}(x) \leq 1$. The value $\mu_{\widetilde{A}^I}(x)$ is the membership degree of x in $\widetilde{A}^I$; $\nu_{\widetilde{A}^I}(x)$ is the non-membership degree of x in $\widetilde{A}^I$; $\mu_{\widetilde{A}^I} : X \to [0,1]$ is the membership function of $\widetilde{A}^I$ in X; and $\nu_{\widetilde{A}^I} : X \to [0,1]$ is the non-membership function of $\widetilde{A}^I$ in X. For each $x \in X$ the value $\pi_{\widetilde{A}^I}(x) = 1 - \mu_{\widetilde{A}^I}(x) - \nu_{\widetilde{A}^I}(x)$ is the degree of hesitancy of $x \in \widetilde{A}^I$.

A triangular intuitionistic fuzzy number (IFN) over the universe of real numbers R, $\widetilde{A}^I \in IFN(R)$ is an special intuitionistic fuzzy set of the real numbers universe whose membership and non-membership functions are defined by:

$$\mu_{\widetilde{A}^I}(x) = \begin{cases} \dfrac{x-\underline{a}}{a-\underline{a}}, & \underline{a} \leq x < a, \\ 1, & x = a, \\ \dfrac{\overline{a}-x}{\overline{a}-a}, & a < x \leq \overline{a}, \\ 0, & \text{otherwise}, \end{cases} \qquad \nu_{\widetilde{A}^I}(x) = \begin{cases} \dfrac{a-x}{a-\underline{b}}, & \underline{b} \leq x < a, \\ 0, & x = a, \\ \dfrac{x-a}{\overline{b}-a}, & a \leq x < \overline{b}, \\ 1, & \text{otherwise}, \end{cases}$$

where $\underline{b}, \underline{a}, a, \overline{a}, \overline{b}$ are real numbers, and $\underline{b} \leq \underline{a} \leq a \leq \overline{a} \leq \overline{b}$. Such a triangular IFN is denoted by $\widetilde{A}^I = (\underline{a}, a, \overline{a}; \underline{b}, a, \overline{b})$.

Basic arithmetic operators over triangular IFNs are defined as follows. For any two triangular IFNs $\widetilde{A}^I = \left(\underline{a}, a, \overline{a}; \underline{b}, a, \overline{b}\right)$ and $\widetilde{C}^I = \left(\underline{c}, c, \overline{c}; \underline{d}, c, \overline{d}\right)$, and any scalar $\lambda \in R$:

1. Addition: $\widetilde{A}^I + \widetilde{C}^I = \left(\underline{a}+\underline{c}, a+c, \overline{a}+\overline{c}; \underline{b}+\underline{d}, a+c, \overline{b}+\overline{d}\right)$;
2. Subtraction: $\widetilde{A}^I - \widetilde{C}^I = \left(\underline{a}-\overline{c}, a-c, \overline{a}-\underline{c}; \underline{b}-\overline{d}, a-c, \overline{b}-\underline{d}\right)$;
3. Scalar multiplication: $\lambda \widetilde{A}^I = \begin{cases} \left(\lambda\underline{a}, \lambda a, \lambda\overline{a}; \lambda\underline{b}, \lambda a, \lambda\overline{b}\right), & \lambda \geq 0, \\ \left(\lambda\overline{a}, \lambda a, \lambda\underline{a}; \lambda\overline{b}, \lambda a, \lambda\underline{b}\right), & \lambda < 0; \end{cases}$
4. Multiplication: $\widetilde{A}^I \cdot \widetilde{C}^I = \left(\underline{e}, a \cdot c, \overline{e}; \underline{f}, a \cdot c, \overline{f}\right)$, where

$$\underline{e} = \min\left\{\underline{a}\cdot\underline{c}, \overline{a}\cdot\overline{c}, \underline{a}\cdot\overline{c}, \overline{a}\cdot\underline{c}\right\}, \overline{e} = \max\left\{\underline{a}\cdot\underline{c}, \overline{a}\cdot\overline{c}, \underline{a}\cdot\overline{c}, \overline{a}\cdot\underline{c}\right\},$$

$$\underline{f} = \min\left\{\underline{b}\cdot\underline{d}, \overline{b}\cdot\overline{d}, \underline{b}\cdot\overline{d}, \overline{b}\cdot\underline{d}\right\}, \overline{f} = \max\left\{\underline{b}\cdot\underline{d}, \overline{b}\cdot\overline{d}, \underline{b}\cdot\overline{d}, \overline{b}\cdot\underline{d}\right\};$$

5. Division: $\widetilde{A}^I/\widetilde{C}^I = \left(\underline{e}, a/c, \overline{e}; \underline{f}, a/c, \overline{f}\right)$, under the assumption that either $\underline{c} > 0$ or $\overline{d} < 0$, where

$$\underline{e} = \min\left\{\underline{a}/\underline{c}, \overline{a}/\overline{c}, \underline{a}/\overline{c}, \overline{a}/\underline{c}\right\}, \overline{e} = \max\left\{\underline{a}/\underline{c}, \overline{a}/\overline{c}, \underline{a}/\overline{c}, \overline{a}/\underline{c}\right\},$$

$$\underline{f} = \min\left\{\underline{b}/\underline{d}, \overline{b}/\overline{d}, \underline{b}/\overline{d}, \overline{b}/\underline{d}\right\}, \overline{f} = \max\left\{\underline{b}/\underline{d}, \overline{b}/\overline{d}, \underline{b}/\overline{d}, \overline{b}/\underline{d}\right\}.$$

2.2 The accuracy function

The score functions of the membership and non-membership functions of the triangular IFN $\widetilde{A}^I = \left(\underline{a}, a, \overline{a}; \underline{b}, a, \overline{b}\right)$ used in [14] are

$$E^{\mu}\left(\widetilde{A}^I\right) = \frac{\underline{a} + \overline{a} + 2a}{4}, S^{\nu}\left(\widetilde{A}^I\right) = \frac{\underline{b} + \overline{b} + 2a}{4},$$

respectively.

The accuracy function of the same triangular IFN $\widetilde{A}^I$ with respect to a parameter $\lambda \in [0,1]$ set by the decision maker is the weighted sum of its score functions, i.e.

$$A^{\lambda}\left(\widetilde{A}^I\right) = \lambda E^{\mu}\left(\widetilde{A}^I\right) + (1-\lambda) S^{\nu}\left(\widetilde{A}^I\right).$$

It is easy to prove that the accuracy function A^{λ} is linear.

The triangular IFN $(a-r, a, a+r; a-s, a, a+s)$, with $0 \le r \le s$, is called symmetric. The above defined accuracy function applied to a symmetric triangular IFN selects its element with largest membership degree, i.e.

$$A^{\lambda}\left((a-r, a, a+r; a-s, a, a+s)\right) = a.$$

In this case the effect of applying the accuracy function is the trivial defuzzification.

Any accuracy function can be used to rank the IFNs. The following three implications were used in [14]: (i) $A^{\lambda}\left(\widetilde{A}^I\right) > A^{\lambda}\left(\widetilde{C}^I\right) \Rightarrow \widetilde{A}^I > \widetilde{C}^I$; (ii) $A^{\lambda}\left(\widetilde{A}^I\right) < A^{\lambda}\left(\widetilde{C}^I\right) \Rightarrow \widetilde{A}^I < \widetilde{C}^I$; (iii) $A^{\lambda}\left(\widetilde{A}^I\right) = A^{\lambda}\left(\widetilde{C}^I\right) \Rightarrow \widetilde{A}^I = \widetilde{C}^I$.

2.3 Intuitionistic fuzzy MOLFP problem

The IFMOLFP problem addressed in [14] is a MOLFP problem that uses triangular IFNs as coefficients of the decision variables in both objective functions and constraints, and also assumes that some of the constraints are fuzzy. Its general mathematical model is given by

$$\text{“}\max\text{''}\left\{Z^I(x) = \left[\widetilde{Z}_1^I(x), \widetilde{Z}_2^I(x), \ldots, \widetilde{Z}_k^I(x)\right] | x \in X^{\text{fuzzy}}\right\}, \tag{1}$$

where $\widetilde{b}^I \in IFN(R)^m$, $\widetilde{a}^I \in IFN(R)^{m\times n}$; $\widetilde{Z}_i^I(\widetilde{x}) = \dfrac{\left(\widetilde{c}_i^I\right)^T x + \widetilde{\alpha}_i}{\left(\widetilde{d}_i^I\right)^T x + \widetilde{\beta}_i} = \dfrac{\widetilde{N}_i^I(x)}{\widetilde{D}_i^I(x)}$ with $\widetilde{c}_i^I, \widetilde{d}_i^I \in IFN(R)^n$, $\widetilde{\alpha}_i^I, \widetilde{\beta}_i^I \in IFN(R)$, for each $i = 1, 2, \ldots, k$;

$$X^{fuzzy} = \left\{x | \widetilde{a_j}^I x \widetilde{\le} \widetilde{b_j}^I, \widetilde{a_p}^I x \le \widetilde{b}_p^I, j \in J, p \in P, x \ge 0\right\}, \tag{2}$$

with J and P denoting the set of indexes for which the corresponding constraints are fuzzy and crisp, respectively. It is assumed that all components of the IFNs $\left(\widetilde{d}_i^I\right)^T x + \widetilde{\beta}_i$ are greater than 0 for each $i = 1, 2, \ldots k$, and the decision variables are crisp.

An intuitionistic fuzzy feasible solution, $\overline{x}^* \in X^{\text{fuzzy}}$ is Pareto optimal to the IFMOLFP problem (1), if there is no other feasible $\overline{x} \in X^{\text{fuzzy}}$ such that $\widetilde{Z}_i^I(\overline{x}) \ge \widetilde{Z}_i^I(\overline{x}^*)$ for each $i \in \{1, \ldots, k\}$, and at least one strict inequality holds. The comparison of IFNs can be interpreted in various ways, but here it is presumed that it is defined with respect to the accuracy function A^{λ}.

2.4 Crisp MOLFP problem

Moges et al. [14] defined the crisp MOLFP problem

$$\text{“ max ”}\left\{\left[Z_1(x), Z_2(x), \ldots, Z_k(x)\right] | x \in X^{\text{crisp}}\right\}, \tag{3}$$

where

$$X^{\text{crisp}} = \left\{x \in R^n | a_j x \tilde{\leq} b_j, a_p x \leq b_p, j \in J, p \in P, x \geq 0\right\}, \tag{4}$$

$b \in R^m$, $a \in R^{m\times n}$ and it still contains some fuzzy constraints; and $Z_i(x) = \dfrac{c_i^T x + \alpha_i}{d_i^T x + \beta_i} = \dfrac{N_i(x)}{D_i(x)}$ with $c_i, d_i \in R^n$, $\alpha_i, \beta_i \in R$, for each $i = 1, 2, \ldots, k$. It is assumed that $d_i^T x + \beta_i > 0$ for each $i = 1, 2, \ldots k$. There is no loss of generality because of this assumption.

The operator ”max” is used with respect to the concept of Pareto optimality. A feasible solution $x^* \in X^{\text{crisp}}$ is said to be an efficient solution (or Pareto optimal) to Problem (3) if and only if there is no $x \in X^{\text{crisp}}$ such that $Z_i(x) \geq Z_i(x^*)$ holds for each $i \in \{1, \ldots, k\}$, and at least one of the inequalities is strict. The feasibility $x \in X^{\text{crisp}}$ has a wider meaning in [14] due to the fuzziness of the constraints indexed with $j \in J$.

The distance function

$$D(\overline{x}^*) = \frac{1}{2k}\sqrt{\sum_{i=1}^{k}\left(Z_i^{\text{ideal}} - Z_i(\overline{x}^*)\right)^2} \tag{5}$$

was used in [14] to quantify the quality of the obtained solution $\overline{x}^*$, and compare the solutions obtained by different approaches from the literature. The values Z_i^{ideal}, $i = 1, \ldots, k$, are the ideal values of the objective functions, i.e.

$$Z_i^{\text{ideal}} = \max\left\{Z_i(x) | x \in X^{\text{crisp}}\right\}, i = 1, \ldots, k. \tag{6}$$

We also employ (5) to measure the quality of the derived solutions.

3 Theoretical findings

The solution approach proposed in [14] has two faulty key points. In this section, we analyze them successively, prove our statements and relate them to relevant results from the literature. In Subsection 3.3, we propose a methodological improvement to the second key point.

3.1 First key point

Step 2 of the approach proposed in [14] assumes that a crisp MOLFP problem can be formulated from the original IFMOLFP problem using the above mentioned accuracy function A^λ, assuring that a Pareto optimal solution to MOLFP problem always yields a Pareto optimal solution to IFMOLFP problem.

The objective functions of MOLFP problem (3) in [14] are constructed such that

$$N_i = A^\lambda\left(\tilde{N}_i^I\right), D_i = A^\lambda\left(\tilde{D}_i^I\right), Z_i = A^\lambda\left(\tilde{Z}_i^I\right), i = 1, \ldots, k, \tag{7}$$

with $\tilde{N}_i^I$, $\tilde{D}_i^I$, and $\tilde{Z}_i^I$ being the IFNs used to define the objective functions of Problem (1).

The definitions given in (7) for $N_i, D_i, Z_i, i = 1, \ldots, k$ are generally in contradiction to the relations $\tilde{Z}_i^I(x) = \frac{\tilde{N}_i^I(x)}{\tilde{D}_i^I(x)}$ and $Z_i(x) = \frac{N_i(x)}{D_i(x)}$, $i = 1, \ldots, k$, between the fractional objective functions and their corresponding numerators and denominators, and the definition of the accuracy function A^λ. Note that for non-negative IFNs coefficients $\tilde{c}_i^I, \tilde{d}_i^I \in IFN(R)^n$ and $\tilde{\alpha}_i^I, \tilde{\beta}_i^I \in IFN(R)$, and positive decision variables we can formalize

$$\tilde{Z}_i^I = \frac{\tilde{N}_i^I}{\tilde{D}_i^I} = \frac{\left(\underline{a}, a, \overline{a}; \underline{b}, a, \overline{b}\right)}{\left(\underline{c}, c, \overline{c}; \underline{d}, c, \overline{d}\right)} = \left(\frac{\underline{a}}{\overline{c}}, \frac{a}{c}, \frac{\overline{a}}{\underline{c}}; \frac{\underline{b}}{\overline{d}}, \frac{a}{c}, \frac{\overline{b}}{\underline{d}}\right). \tag{8}$$

Then, on one side

$$Z_i = A^\lambda\left(\widetilde{Z}_i^I\right) = A^\lambda\left(\left(\frac{\underline{a}}{\overline{c}}, \frac{a}{c}, \frac{\overline{a}}{\overline{c}}; \frac{\underline{b}}{\overline{d}}, \frac{a}{c}, \frac{\overline{b}}{\overline{d}}\right)\right) = \frac{\lambda}{4}\left(\frac{\underline{a}}{\overline{c}} + \frac{\overline{a}}{\underline{c}} + 2\frac{a}{c}\right) + \frac{1-\lambda}{4}\left(\frac{\underline{b}}{\overline{d}} + \frac{\overline{b}}{\underline{d}} + 2\frac{a}{c}\right), \quad (9)$$

and on the other side

$$Z_i = \frac{N_i}{D_i} = \frac{A^\lambda\left(\widetilde{N}_i^I\right)}{A^\lambda\left(\widetilde{D}_i^I\right)} = \frac{A^\lambda\left(\left(\underline{a}, a, \overline{a}; \underline{b}, a, \overline{b}\right)\right)}{A^\lambda\left(\left(\underline{c}, c, \overline{c}; \underline{d}, c, \overline{d}\right)\right)} = \frac{\frac{\lambda}{4}\left(\underline{a} + \overline{a} + 2a\right) + \frac{1-\lambda}{4}\left(\underline{b} + \overline{b} + 2a\right)}{\frac{\lambda}{4}\left(\underline{c} + \overline{c} + 2c\right) + \frac{1-\lambda}{4}\left(\underline{d} + \overline{d} + 2c\right)}. \quad (10)$$

It is obvious that the expression given for Z_i in (9) cannot be generally equal to the expression given in (10). In the following, we exemplify the discrepancy between $A^\lambda\left(\frac{\widetilde{N}^I}{\widetilde{D}^I}\right)$ and $\frac{A^\lambda\left(\widetilde{N}^I\right)}{A^\lambda\left(\widetilde{D}^I\right)}$ for $\widetilde{N}^I = (2, 4, 5; 1, 4, 6)$ and $\widetilde{D}^I = (1, 2, 4; 1, 2, 5)$: $\frac{\widetilde{N}^I}{\widetilde{D}^I} = (0.5, 2, 5; 0.2, 2, 6)$; $A^\lambda\left(\widetilde{N}^I\right) = 3.75\lambda + 3.75\left(1-\lambda\right) = 3.75$; $A^\lambda\left(\widetilde{D}^I\right) = 2.25\lambda + 2.5\left(1-\lambda\right) = 2.5 - 0.25\lambda$. Consequently,

$$\frac{A^\lambda\left(\widetilde{N}_i^I\right)}{A^\lambda\left(\widetilde{D}_i^I\right)} = \frac{3.75}{2.5 - 0.25\lambda}; \quad A^\lambda\left(\frac{\widetilde{N}^I}{\widetilde{D}^I}\right) = 2.375\lambda + 2.55\left(1-\lambda\right) = 2.55 - 0.175\lambda.$$

The above contradiction is an argument for claiming that the statement of Theorem 3.1 given in [14] is not correct. Theorem 3.1 [14] affirms that a Pareto-optimal solution to the crisp MOLFP problem (3) is a Pareto-optimal solution to the IFMOLFP problem (1).

The proof of Theorem 3.1 given in [14] has two flaws: (i) deriving $\frac{A^\lambda(N_i(\overline{x}))}{A^\lambda(D_i(\overline{x}))} \geq \frac{A^\lambda(N_i(\overline{x}^*))}{A^\lambda(D_i(\overline{x}^*))}$ from $\frac{N_i(\overline{x})}{D_i(\overline{x})} \geq \frac{N_i(\overline{x}^*)}{D_i(\overline{x}^*)}$, instead of deriving $A^\lambda\left(\frac{N_i(\overline{x})}{D_i(\overline{x})}\right) \geq A^\lambda\left(\frac{N_i(\overline{x}^*)}{D_i(\overline{x}^*)}\right)$; and (ii) concluding $\nexists \overline{x} \in S^I$ form $\nexists \overline{x} \in S^c$ and $S^c \subseteq S^I$.

The construction of Problem (3) from the intuitionistic fuzzy Problem (1) can be completed only through the trivial defuzzification, using the core selection criterion, namely replacing each triangular intuitionistic fuzzy coefficient $\widetilde{A}^I$ by its element with largest degree of membership, i.e. using the most simple accuracy function $Acc\left(\left(\underline{a}, a, \overline{a}; \underline{b}, a, \overline{b}\right)\right) = a$. Only in this very particular case $\frac{Acc\left(\widetilde{N}_i^I\right)}{Acc\left(\widetilde{D}_i^I\right)} = Acc\left(\frac{\widetilde{N}^I}{\widetilde{D}^I}\right)$. For their numerical examples, Morges et al. [14] used symmetric triangular intuitionistic fuzzy numbers $\widetilde{A}^I_{sym}$ for all coefficients, and further exploited the particular case $A^\lambda\left(\widetilde{A}^I_{sym}\right) = Acc\left(\widetilde{A}^I_{sym}\right)$.

We addressed this key point aiming to clarify the border between the defuzzifications made in accordance to true theoretical statements, and those made for modeling reasons. The second kind of defuzzification is generally made in accordance to particularities of real-life problems that have to be solved, by a modeler that has a high level of expertise and is familiar with all problem specifications. Any mathematical model built do describe a real-life problem has a certain level of inaccuracy, thus trying to prove its accuracy is a mistaken view point.

3.2 Second key point

Step 3 of the approach proposed in [14] is based on the linearization of the crisp MOLFP problem using the misleading transformation proposed in [3].

The well known Charnes-Cooper transformation [4] transforms a single objective LFP problem to an equivalent LP problem, assuring that an optimal solution to the constructed LP problem yields an optimal solution to the LFP problem. However, the transformation proposed in [3] formally extends the Charnes-Cooper transformation to the multiple objective case but it is not able to conserve the Pareto optimality. As it was already proved in [24], a Pareto optimal solution to the multiple objective LP problem is not necessary a Pareto optimal solution to the MOLFP problem.

As a direct consequence, Theorem 3.2 given in [14] is incorrect, and its proof is inaccurate since r is used in the transformation as equal to $\frac{1}{D_i(x)}$ for a fixed index $i = 1, \ldots, k$, while it is replaced discretionary by any $\frac{1}{D_t(x)}$, $t = 1, \ldots, k$ whenever it is beneficial.

3.3 Methodological improvement

To overcome the deficiency of the second key point, we propose a procedure able to derive a Pareto optimal solution to the MOLFP problem exploiting the good characteristics of the Pareto optimal solution to the MOLFP problem derived using the weighted intuitionistic fuzzy goal programming adopted in [14].

Starting from the solution derived by Moges et al. [14], the fuzzy constraints and the objective values are evaluated and then a new family of linear fractional optimization Models (11) is proposed to derive the desired Pareto optimal solution to the MOLFP problem.

$$\begin{array}{ll} \max & \dfrac{N_h(x)}{D_h(x)} \\ \text{s.t.} & \\ & a_j x \le b_j, \qquad j \in J, \\ & a_p x \le b_p, \qquad p \in P, \\ & N_i(x) \ge \frac{N_i(\overline{x}^*)}{D_i(\overline{x}^*)} D_i(x), \quad i \in \{1,2,\ldots,k\} \setminus \{h\}. \\ & x \ge 0, \end{array} \tag{11}$$

Models (11) separately use the objective functions of the initial crisp MOLFP problem (3), and they are solved successively for $h = 1, \ldots, k$ in the lexicographic manner. For $h = 1$, $\overline{x}^*$ is the solution derived by Moges et al.'s procedure [14], and for $h > 1$, $\overline{x}^*$ is the solution derived in Step $h - 1$. Note that instead of the fuzzy constraints

$$a_j x \widetilde{\le} b_j, j \in J \tag{12}$$

from X^{crisp}, we used their full achievement from,

$$a_j x \le b_j, j \in J, \tag{13}$$

in Model (11).

In the case of infeasibility, the constraints (13) can be relaxed, inevitably reducing the satisfaction level of the decision maker. In order to assure that the new derived solution provides the same or better satisfaction/acceptance degree, and the same or smaller dissatisfaction/rejection degree as $\overline{x}^*$, the constraints (13) should be replaced by

$$a_j x \le b_j^*, j \in J, \tag{14}$$

where b_j^*, $j \in J$ are the values of the left-hand-side expressions of the fuzzy constraints evaluated at $\overline{x}^*$.

The above methodology is summarized in Algorithm 1. Basically, this algorithm describes the lexicographic method - well know in multiple objective optimization - adapted to linear fractional objective functions.

There are two overall advantages of applying the lexicographic procedure of solving Models (11) successively: (i) the value of the distance function decreases, since the values of all objective functions either increase or remain the same; and (ii) the derived solution is Pareto optimal, that is highly required when solving any multiple objective optimization problem.

4 Illustrative example

To illustrate our theoretical statements we recall the Agricultural Land Allocation Problem (ALAP) solved by Moges et al. in [14]. The ALAP problem introduced in [14] was originally formulated as an IFMOLFP problem with symmetric triangular IFN coefficients. Applying the accuracy function, each coefficient was defuzzificated, thus being replaced by its element with maximal membership degree.

Algorithm 1 Lexicographic improvement

Require: The matrix/vector parameters $\widetilde{a}$, $\widetilde{b}$, $\widetilde{c}$, $\widetilde{d}$, $\widetilde{\alpha}$ and $\widetilde{\beta}$ that are the IFN coefficients of Problem (1); the set of indexes J and P that make distinction between fuzzy and crisp constraints; the permutation σ describing the importance of the criteria (i.e. $\sigma(i)$ is the position of function Z_i with respect to the order of importance); $\overline{x}^*$ the solution derived by Moges et al. [14].

1: **for** $k = \overline{1,p}$ **do**
2: Solve Problem (11) for the index $h = \sigma(k)$ replacing the constraints $N_{\sigma(j)}(x) \geq \frac{N_{\sigma(j)}(\overline{x}^*)}{D_{\sigma(j)}(\overline{x}^*)} D_{\sigma(j)}(x)$ by $z_{\sigma(j)}(x) \geq z_{\sigma(j)}, \forall j \in \{1, 2, \ldots, k\}$.
3: **if** the problem is infeasible **then**
4: stop and declare that there is no improved solution.
5: **else**
6: put the value of the optimal solution in $z_{\sigma(j)}$ and continue.
7: **end if**
8: **end for**

Ensure: The final solution derived for $h = \sigma(p)$.

Table 1: The defuzzified objective functions derived by Moges et al. [14] to ALAP problem

$Z_1(x)$	$\frac{31070.21x_1 + 71969x_2 + 32350x_3 + 13241x_4 + 13500x_5 + 14148.6x_6 + 39834.64x_7 + 13445x_8 + 31874x_9}{12847.12x_1 + 16520.2x_2 + 14325x_3 + 10534.97x_4 + 10044x_5 + 9500x_6 + 13254.02x_7 + 11265x_8 + 11780x_9}$
$Z_2(x)$	$\frac{97.5x_1 + 115.55x_2 + 111.03x_3 + 62.64x_4}{x_1 + x_2 + x_3 + x_4}$
$Z_3(x)$	$\frac{72.92x_5 + 37.04x_6 + 84.76x_7 + 44.85x_8 + 61.81x_9}{x_5 + x_6 + x_7 + x_8 + x_9}$

As a consequence, Moges et al. [14] solved the following crisp model with several fuzzy constraints:

$$\begin{array}{ll} \text{"max"} & \{Z_1(x), Z_2(x), Z_3(x)\} \\ \text{s.t.} & \\ & 111.03x_3 \tilde{\geq} 45, \\ & 44.85x_8 \tilde{\geq} 15, \\ & x_1 + x_2 + x_3 + x_4 \leq 3.5, \\ & x_5 + x_6 + x_7 + x_8 + x_9 \leq 3.5, \\ & 75x_1 + 125x_2 + 119x_3 + 60x_4 \tilde{\leq} 110, \\ & 89x_5 + 49x_6 + 111x_7 + 96x_8 + 85x_9 \tilde{\leq} 110, \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0, \end{array} \tag{15}$$

where the objective functions Z_1, Z_2, Z_3 are described in Table 1. They derived the solution

$$\overline{x}^* = \left(\begin{array}{ccccccccc} 0.004193, & 1.1656683, & 2.330139, & 0, & 2.351677, & 0, & 0, & 0.160962, & 0 \end{array} \right).$$

to ALAP problem (15).

We construct the particular form of Problem (11) that corresponds to Problem (15) and to its solution $\overline{x}^*$. We set $h = 1$, and using the values $Z_2(\overline{x}^*) = 112.51917$ and $Z_3(\overline{x}^*) = 71.12181$ as lower bounds on the objective functions Z_2 and Z_3, respectively.

It means that the inequalities

$$Z_2(x) \geq 112.51917, Z_3(x) \geq 71.12181 \tag{16}$$

further provide the constraints

$$\frac{97.5x_1 + 115.55x_2 + 111.03x_3 + 62.64x_4}{x_1 + x_2 + x_3 + x_4} \geq 112.51917, \tag{17}$$

and
$$\frac{72.92x_5 + 37.04x_6 + 84.76x_7 + 44.85x_8 + 61.81x_9}{x_5 + x_6 + x_7 + x_8 + x_9} \geq 71.12181, \tag{18}$$
to be added to the constraint system.

Since both constraints in (18) can be easily linearized, we proceed with solving the single objective LFP Problem (19):

$$\begin{array}{ll} \max & Z_1(x) \\ \text{s.t.} & \\ & 111.03x_3 \geq 45, \\ & 44.85x_8 \geq 15, \\ & x_1 + x_2 + x_3 + x_4 \leq 3.5, \\ & x_5 + x_6 + x_7 + x_8 + x_9 \leq 3.5, \\ & 75x_1 + 125x_2 + 119x_3 + 60x_4 \leq 110, \\ & 89x_5 + 49x_6 + 111x_7 + 96x_8 + 85x_9 \leq 110, \\ & 97.5x_1 + 115.55x_2 + 111.03x_3 + 62.64x_4 \geq 112.51917(x_1 + x_2 + x_3 + x_4) \\ & 72.92x_5 + 37.04x_6 + 84.76x_7 + 44.85x_8 + 61.81x_9 \geq 71.12181(x_5 + x_6 + x_7 + x_8 + x_9) \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0, \end{array} \tag{19}$$

and derive the solution
$$(\overline{x}^*)^1 = \begin{pmatrix} 0, & 0.494158, & 0.405296, & 0, & 0, & 0, & 0.701739, & 0.334448, & 0 \end{pmatrix}.$$

We can use $(\overline{x}^*)^1$ to negate the Pareto optimality of $\overline{x}^*$, and show the effectiveness of the methodological extension that we proposed in Section 3.3., as follows. The values of the objective functions at $\overline{x}^*$ and $(\overline{x}^*)^1$ are
$$\begin{aligned} Z(\overline{x}^*) &= \begin{pmatrix} 2.47446, & 112.51917, & 71.12181 \end{pmatrix}, \\ Z\left((\overline{x}^*)^1\right) &= \begin{pmatrix} 3.00044, & 113.51328, & 71.87833 \end{pmatrix}; \end{aligned}$$
and the fulfillment of the fuzzy constraints for $\overline{x}^*$ and $(\overline{x}^*)^1$ are
$$\begin{aligned} a\overline{x}^*\left(\tilde{\geq}, \tilde{\leq}\right) b: &\quad 258.71533317\tilde{\geq}45, \quad 7.2191457\tilde{\geq}15, \quad 423.309554\tilde{\leq}110, \quad 224.751605\tilde{\leq}110, \\ a(\overline{x}^*)^1\left(\tilde{\geq}, \tilde{\leq}\right) b: &\quad 45.0000149\tilde{\geq}45, \quad 14.9999928\tilde{\geq}15, \quad 109.999974\tilde{\leq}110, \quad 110.000037\tilde{\leq}110. \end{aligned}$$

It is clear that $(\overline{x}^*)^1$ yields better values than $\overline{x}^*$ for all three objective functions; and fully satisfy all fuzzy constraints, hence assuring the decision maker's satisfaction level equal to 1, while $\overline{x}^*$ fully satisfies only the first one. In other words, our solution $(\overline{x}^*)^1$ dominates $\overline{x}^*$, and as a direct consequence the distance function $D(\overline{x})$ is smaller at $(\overline{x}^*)^1$ than at $\overline{x}^*$. The exact values are reported in Table 2.

The solution
$$\hat{\overline{x}} = \begin{pmatrix} 2.17642, & 0.776409, & 0.208268, & 0, & 1.996459, & 0, & 0, & 0.041792, & 0 \end{pmatrix},$$

obtained by applying other two methods (Angelov [1] and Zimmerman [28]) to the same Problem (15) was reported in [14].

Aiming to obtain a better solution than $\hat{\overline{x}}$, we solve Problem (20)

$$\begin{array}{ll} \max & Z_1(x) \\ \text{s.t.} & \\ & 111.03x_3 \geq 45, \\ & 44.85x_8 \geq 14.5, \\ & x_1 + x_2 + x_3 + x_4 \leq 3.5, \\ & x_5 + x_6 + x_7 + x_8 + x_9 \leq 3.5, \\ & 75x_1 + 125x_2 + 119x_3 + 60x_4 \leq 110, \\ & 89x_5 + 49x_6 + 111x_7 + 96x_8 + 85x_9 \leq 110.6, \\ & 97.5x_1 + 115.55x_2 + 111.03x_3 + 62.64x_4 \geq 112.51917(x_1 + x_2 + x_3 + x_4) \\ & 72.92x_5 + 37.04x_6 + 84.76x_7 + 44.85x_8 + 61.81x_9 \geq 72.3445(x_5 + x_6 + x_7 + x_8 + x_9) \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0. \end{array} \tag{20}$$

Table 2: The values of the objective functions at different solutions obtained in the literature or derived by solving the new proposed models.

	Ideal values	Methods [1], [28]	Method [14]	Our improvements	
$\overline{x}$	-	$\hat{\overline{x}}$	$\overline{x}^*$	$(\overline{x}^*)^1$	$(\overline{x}^*)^2$
$Z_1(\overline{x})$	3.3602	2.453591	2.474461	3.000441	3.008005
$Z_2(\overline{x})$	113.9153	102.824749	112.519168	113.513278	113.513278
$Z_3(\overline{x})$	75.533	72.344456	71.121809	71.878329	72.344490
$D(\overline{x})$	0	0.929001	0.785114	0.615712	0.538833

that is similar to Problem (19) but it uses the values $Z_2\left(\hat{\overline{x}}\right)$ and $Z_3\left(\hat{\overline{x}}\right)$, namely 102.824749 and 72.344456, as bounds to Z_2 and Z_3, respectively; and the relaxed fuzzy constraints

$$44.85x_8 \geq 14.55, 89x_5 + 49x_6 + 111x_7 + 96x_8 + 85x_9 \leq 110.6,$$

instead of

$$44.85x_8 \geq 15, 89x_5 + 49x_6 + 111x_7 + 96x_8 + 85x_9 \leq 110,$$

respectively. In this way we obtain the solution

$$(\overline{x}^*)^2 = \left(0, \quad 0.494158, \quad 0.405296, \quad 0, \quad 0, \quad 0, \quad 0.715957, \quad 0.3233, \quad 0 \right),$$

that yields the objective values reported in Table 2. From the point of view of the fuzzy constraints fulfillment, i.e. from

$$a\hat{\overline{x}}\left(\tilde{\geq},\tilde{\leq}\right)b: \quad 23.123996\tilde{\geq}45, \quad 1.874371\tilde{\geq}15, \quad 285.066517\tilde{\leq}110, \quad 181.696883\tilde{\leq}110$$

and

$$a\left(\overline{x}^*\right)^2\left(\tilde{\geq},\tilde{\leq}\right)b: \quad 45.0000149\tilde{\geq}45, \quad 14.500005\tilde{\geq}15, \quad 109.999974\tilde{\leq}110, \quad 110.508027\tilde{\leq}110$$

we conclude that $(\overline{x}^*)^2$ is better than $\hat{\overline{x}}$. It is also better than $\overline{x}^*$ and worse than $(\overline{x}^*)^1$.

A summary of our numerical results compared to the results reported in the literature can be seen in Table 2. The distances of the obtained results from the ideal values are also reported in the same table. The graphic representations of their corresponding normalized values (see Table 3) are shown in Figure 1.

Comparing the solutions derived by our extended methodology we may conclude that $(\overline{x}^*)^2$ is better than $(\overline{x}^*)^1$ since its distance from the ideal values is smaller; but in the same time, $(\overline{x}^*)^1$ can be considered better than $(\overline{x}^*)^2$ since it fully satisfies the crisp constraints.

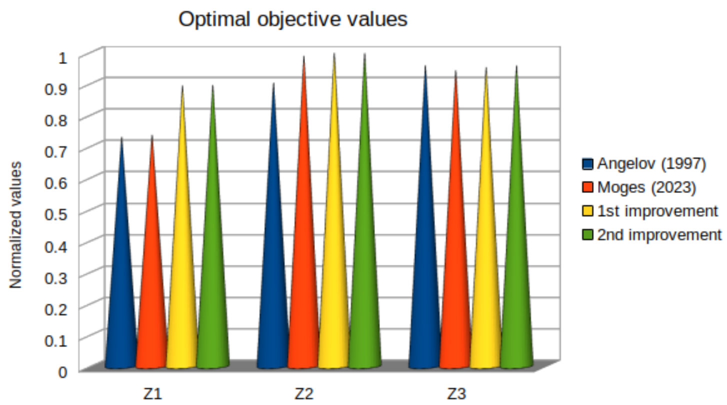

Figure 1: Comparative graphic representations of the normalized values reported in Table 3

Table 3: The normalized values of the objective functions at different solutions obtained in the literature or derived by solving the new proposed models with respect to the ideal values.

	Methods [1], [28]	Method [14]	Our improvements	
$\overline{x}$	$\hat{\overline{x}}$	$\overline{x}^*$	$(\overline{x}^*)^1$	$(\overline{x}^*)^2$
$Z_1(\overline{x})$	0.73	0.74	0.89	0.89
$Z_2(\overline{x})$	0.90	0.99	1.00	1.00
$Z_3(\overline{x})$	0.96	0.94	0.95	0.96

5 Final conclusion

The methodological improvement introduced in Section 3.3 and illustrated in Section 4 is able to derive a Pareto optimal solution to MOLFP problems, thus overcoming one deficiency of Moges et al.'s solution approach [14] to IFMOLFP problem, reaching in the same time the general requirement of any procedure to solve multiple objective optimization problems.

The transformation of an intuitionistic fuzzy problem into an equivalent crisp one using an accuracy function is still an issue. As pointed out in the end of Section 3.1., such transformation is generally made by the modeler that has a high level of expertise and is familiar with all problem specifications, and in accordance to the particularities of real-life problems that have to be solved. Justifications should be related to modeling reasons, therefore they do not need any theoretical prove of preserving the equivalence.

To remove the inconsistencies arisen from the defuzzification of optimization problems with fuzzy coefficients through various ranking functions, the idea of using the extension principle on the level of the solution concept, and not only to perform fuzzy arithmetic, is advanced, in the recent literature.

Diniz et al. [6] addressed the optimization of fuzzy-valued functions generally providing the framework for a wider use of the extension principle; while Stanojević [23] used the extension principle to empirically derive the Pareto front of a fully fuzzified MOLFP problem. Applying this general idea to solve IFMOLFP problems is one direction for further researches in this area. Using new operators to aggregate the intuitionistic fuzzy quantities recently introduced in the literature; or solving similar problems using Z-numbers [27] are other possible research directions.

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Author contributions

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Conflict of interest

The authors declare no conflict of interest.

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